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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

EDITED BY

GEORGE E. HALE
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MAY 1922

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VOLUME LV MAY 1922 NUMBER 4

THE QUANTUM THEORY OF BAND SPECTRA AND ITS APPLICATION TO THE DETERMINATION OF TEMPERATURE

By RAYMOND T. BIRGE

ABSTRACT

Structure and theory of spectrum bands.—(1) *A summary of recent work* by Fortrat, Heurlinger, Kratzer, and others is given. It is now known that bands in the visible have the same structure as infra-red bands, each starting from a line of zero intensity and increasing in intensity symmetrically in both directions to maxima, the separation of the lines in one branch meanwhile decreasing until it becomes zero and then negative, the branch reaching a head. It is shown that Kemble's formula, $m^2_{max} = 0.699T/\Delta\nu_0$, for the number of the line of maximum intensity, m_{max}, as a function of absolute temperature and limiting separation of successive lines for $m = +1$, a formula based on the statistical distribution of angular velocity given by the quantum theory, may be expected to give fairly accurate values of T though it neglects some factors. (2) *Evidence* is presented to show *that Kemble's formula holds* not only for infra-red bands but also for the CN band λ 3883.

Cyanogen band λ 3883.—(1) *Position of maximum intensity as function of temperature.* From a study of spectrograms made by Strutt and Fowler and by King, m_{max} is found to vary from 9 ± 2 for light from a discharge tube containing active nitrogen at about 100°C, to 23 ± 2 for light from a furnace at 2500° to 2700°C, while the values corresponding to these temperatures as computed from Kemble's formula are 8.4, 22.9, and 23.7 respectively. This agreement justifies us in using the formula to determine the approximate temperature in the case of arcs, the sun, and similar sources which emit or absorb these bands. (2) *The intensity distribution in the A series from a 13-ampere arc* is symmetrical and corresponds to the empirical formula $\log I = 1.5 \log m - c_2 m^{1.5} + \text{const.}$ (3) *The relative intensity of the A, B, C, and D series* seems to be *independent of the temperature.*

Spectroscopic determination of temperature, using CN bands.—From the position of maximum intensity on spectrograms made by the author, the *temperature of a 13-ampere arc* was computed to be $4500°C \pm 300$, while from King's spectrograms *the temperature of a 4-ampere arc* comes out $3900°C \pm 300$. By comparing solar spectrograms taken by St. John with the arc spectrograms the *temperature of the reversing layer of the sun* comes out $4000°C \pm 500$.

INTRODUCTION

King[1] has recently obtained spectrograms of the λ 3883 CN band[2] in the electric furnace, at temperatures of 2300° C., 2500° C., and 2700° C. It seemed to the author that these spectrograms furnished valuable material for the quantitative testing of a number of points in connection with the quantum theory of band spectra, and Dr. King has kindly loaned for this purpose all his original plates. He furnished also an absorption spectrogram of the λ 3883 band, taken in the furnace with the plug at about 3500° C. All of these plates were taken in the third order of the 30-foot plane grating of the Pasadena laboratory, and have a dispersion of 1.73 mm per A. The author is indebted also to Dr. St. John, of the Mount Wilson Observatory, for a solar spectrogram of this band, having a dispersion of 1.76 mm per A. I have, in addition, a series of spectrograms of the λ 3883 band, as emitted by the carbon arc, taken in the third order of the 21-foot concave grating of the University of Wisconsin (dispersion 1.13 mm per A). Enlargements, of varying intensity, were made of all of these plates, on a uniform scale of 1.05 cm per A (the scale of King's published enlargement). These and numerous direct contact prints have furnished the chief material for a study of the intensity relations in this band. Additional quantitative wave-length measurements were made from the original negatives, using the eight-inch Gaertner comparator for which the author is indebted to the Rumford Committee of the American Academy of Sciences. Two preliminary reports on this work have been made to the American Physical Society.[3] The present paper is concerned mainly with the results submitted in the later report.

The recent important development of the quantum theory of band spectra, as applied particularly to diatomic molecules, was initiated independently by Heurlinger[4] and by Lenz.[5] Further

[1] *Astrophysical Journal*, **53**, 161, 1921.

[2] Theoretical considerations indicate that this band is due to nitrogen, but its origin is still a matter of dispute (see Barratt, *Proceedings Royal Society* [A], **98**, 40, 1920, and Holst and Oosterhuis, *K. Akad. Amsterdam, Proc.*, **23**, 5, 727–728, 1921).

[3] Berkeley, August, 1921 (Abstract, *Physical Review*, **18**, 319, 1921) and Toronto, December, 1921.

[4] *Physikalische Zeitschrift*, **20**, 188, 1919.

[5] *Verhandlungen der deutschen physikalischen Gesellschaft*, **21**, 632, 1919.

developments, of use in the present investigation, have been made by Heurlinger[1] and by Kratzer.[2] In the case of ordinary bands, such as the λ 3883 band, the empirical formulation of the experimental material is due chiefly to Heurlinger,[3] who elaborated an idea first suggested by Fortrat.[4] The results are briefly as follows.

There is no essential difference in the structure of near infra-red bands, such as the halogen acid bands investigated by Imes[5] and by Colby and Meyer,[6] and many ordinary bands in the visible and ultra-violet, such as the λ 3883 CN band, investigated in great detail by Uhler and Patterson,[7] Birge,[8] Heurlinger,[9] Bachem,[10] and many previous workers. Each true "band" consists of one "series" having two similar branches. The common origin of these two branches is a line of zero intensity, and is labeled $m=0$. From this point the positive branch (Heurlinger's "P" branch) runs toward the longer wave-lengths and successive lines are designated $m=+1, +2$, etc. The negative branch (Heurlinger's "R" branch) runs in the opposite direction and the lines are designated $m=-1, -2$, etc. Successive lines are not equally spaced but crowd closer together in one direction, until the series presently reaches a turning-point, where the separation of successive lines has become zero, and then turns back on itself. This turning point is what has always been known as the "head" of a band. Either the positive or the negative branch may thus invert. In the former case, illustrated by the λ 3883 CN band, the band is said to be degraded toward the violet, while in the latter case, illustrated by the halogen acid infra-red bands, the band is said to be degraded toward the red.

The two types of bands, ultra-violet and infra-red, do not appear similar in structure, due mainly to the fact that the infra-red bands have not been followed to the "head" because of the rapid decrease in intensity of the lines. But this in turn is due

[1] *Zeitschrift für Physik*, **1**, 82, 1920.

[2] *Ibid.*, **3**, 289, 1920.

[3] Dissertation, Lund, 1918.

[4] Thesis, Paris, 1914.

[5] *Astrophysical Journal*, **50**, 251, 1919.

[6] *Ibid*, **53**, 300, 1921.

[7] *Ibid.*, **42**, 434, 1915.

[8] *Ibid.*, **46**, 85, 1917; *Physical Review*, **11**, 136; 1918.

[9] Dissertation, Lund, 1918.

[10] *Zeitschrift für Physik*, **3**, 372, 1920.

merely to the low temperature of the absorbing molecules, in the experimental conditions employed. Theoretically, each line of a band series corresponds to a different change of molecular rotation. The entire series corresponds to the same change of atomic vibratory motion for the near infra-red bands, or to a simultaneous change in atomic vibratory and electronic rotatory motion, in the case of visible and ultra-violet bands. The exact relation between the labeling of the lines (value of m) and the number of quantum units of angular momentum was in doubt until the work of Kratzer (*loc. cit.*). His conclusions, which seem to the author to be entirely valid, are that the missing line ($m=0$) indicates, in emission, the transition from one to zero units of angular momentum ($1\rightarrow 0$). In absorption it indicates the reverse transition $0\rightarrow 1$. The positive branch always indicates, in emission, an increase in momentum, while the negative indicates a decrease. The relations are reversed in the case of absorption. The value of m, for any line, thus indicates in the case of absorption, the initial number of units of angular momentum possessed by the molecule, while in the case of emission it indicates the final number. Thus in emission $m=+60$ indicates the transition $59\rightarrow 60$, $m=-59$ indicates the transition $60\rightarrow 59$.

In the case of all visible and ultra-violet bands, it has been customary to consider a series as starting at the point for which $\Delta\nu=0$, where a strong head generally appears, and the lines are numbered from this point. The numbers thus obtained will be indicated, in the present article, by m', and in the case of the λ 3883 band, the actual numbers for the A series are those used by Uhler and Patterson (*loc. cit.*). The idea that a series has two branches, centering at a point other than the "head" of the band, is due to Fortrat (*loc. cit.*) who found that the lines showing irregularities of position (the "perturbations") could thus be arranged in pairs at $\pm m$. Heurlinger found that by taking for this new origin the point of minimum intensity, similar perturbations occurred for the lines[1] $+m$ and $-(m-1)$. The reason for this

[1] Heurlinger in his dissertation had not yet theoretically interpreted the two branches. He seems to have adopted the policy of calling the branch which returned on itself the negative branch, and gave the lines of this branch negative values of m. This is theoretically correct for all bands degraded to the red, but is incorrect for all

discrepancy of one unit does not seem to have been explained, but the explanation follows immediately from Kratzer's hypothesis and is an important verification of the truth of this hypothesis. Thus the lines $+60$ and -59 show the same perturbation, and as previously noted these two lines indicate transitions between the same two rotational energy states (59 and 60) but in opposite directions. There is evidently an irregularity in this transition which affects the emitted and absorbed frequency. The experimental material at hand has furnished new data on the quantitative relations, in the case of perturbations, but this matter is reserved for a separate article. Suffice it to say here that Bachem's results (*loc. cit.*) for the perturbations, from magnetic effects, are independently confirmed by King's furnace plates, while the similarity of corresponding perturbations is even greater than has been supposed.

Heurlinger found that by taking the origin of a band series at the point of minimum intensity, the two "branches" of the series show qualitatively a similar intensity-curve. Each branch begins at zero intensity, rises to a maximum, and then decreases. Heurlinger also called attention to the fact that the intensity distribution in at least one branch changed with physical conditions and could be interpreted as a change due to temperature. In the case of near infra-red bands, each branch changes with temperature, the point of maximum intensity moving toward the higher m's, as the temperature increases. The main purpose of the present investigation has been to make a quantitative study of this matter for the λ 3883 band.

The theoretical material on this point is as follows. The intensity[1] of an absorption line (in band spectra) depends on three factors, i.e.,

$$I = (E_{m'} - E_m) B^m_{m'} a_m \qquad (1)$$

bands degraded to the violet. In agreement with most other investigators such as Bachem (*loc. cit.*) the author in his August, 1921, report used the original Heurlinger nomenclature for the λ 3883 band. But in the present paper the theoretically correct nomenclature is used, thus reversing the sign of m, and the designation of the two branches. In the old nomenclature, the rule for equal perturbations reads $+m$ and $-(m+1)$.

[1] Heurlinger, *Zeitschrift für Physik*, 1, 82, 1920.

where the transition is from m to m' units of angular momentum. The first factor (energy change) is proportional to the emitted frequency and changes by only 6 per cent during the course of the longest (A) series of the λ 3883 band. The second factor gives the probability of a transition from m to m', under the influence of absorbed radiation of the proper frequency. Little is known concerning this factor except that, from experimental data, it changes rather slowly in value with m, if m is not too small. It is also probable that

$$B^m_{m+1} = B^m_{m-1}.$$

The third factor, a_m, gives the number of molecules, at any instant, possessing m units of angular momentum. A statistical distribution of angular momentum is to be expected, on either classical or quantum theory, and such a distribution was computed first by Kemble,[1] who applied it to the 4.7 μ CO infra-red band, on the assumption that the intensity depended on this one factor alone. Kemble, as a matter of fact, worked out the distribution of angular velocity, and accordingly the results hold for angular momentum only in case J = constant, where J refers to the initial moment of inertia, in the case of absorption. This does not exclude a possible change in the moment of inertia (from J to J') during the transition, due presumably to the change in vibration and electronic configuration. What is required is that J and J' be independent of the amount of molecular rotational energy. If $J = J'$ the lines of a series would have constant frequency difference. If J = constant, and J' = another constant, then Deslandres' parabolic law should be true.[2] But Deslandres' formula is probably not accurately true for any known band,[3] including infra-red bands, and this indicates that J (and presumably J') both vary slowly with m. No attempt has yet been made to interpret the variation quantitatively in terms of J and J'.

Thus Kemble's formula is not a true expression for a_m, but probably approximates it very closely. The value of m correspond-

[1] *Physical Review* (2), **8**, 689, 1916.

[2] See Sommerfeld's *Atombau* (second edition), pp. 550–566, for a summary of the work underlying these conclusions.

[3] Birge, *Physical Review* (2), **13**, 360, 1919.

ing to the line of maximum intensity is given, in Kemble's formula, by

$$m_{\max}=\frac{2\pi}{h}\sqrt{kJT}. \qquad (2)$$

where h = Planck's constant (6.55×10^{-27} erg-sec.), k = gas constant per molecule (1.372×10^{-16}erg/degree) and T = absolute temperature. If J is constant, its value can be accurately obtained from the limiting separation of successive lines ($\Delta\nu$) for $m=+1$, in which case

$$\Delta\nu_0=h/4\pi^2J.^1$$

Hence it is possible to compute the temperature of the absorbing molecules, from the value of m for the line of maximum intensity, in either branch of a band series. Heurlinger (dissertation, p. 65) obtained this result, but considered that from experimental evidence only qualitative values of temperature were to be expected, in the case of ordinary band spectra. It will, however, be shown in this article that from recent experimental evidence the various approximations made must have relatively little effect, so that a close estimate of temperature is possible. Since in the case of discontinuous emission and absorption by vapors, no other method of quantitative temperature determination is as yet known, any results, even if approximate, should be of value in both terrestrial and astronomical work.

A formula similar to (1) applies to the case of emission, but the explanation of the expected value of the various factors is more involved. It therefore seems simpler to use the fact of the known

[1] In Sommerfeld's *Atombau* (p. 555, 2d edition) J' denotes the true moment of inertia (i.e., that corresponding to the initial state in the case of absorption, or to the final state in the case of emission). Sommerfeld's equations then indicate that J' is to be obtained from the limiting $\Delta\nu$ for $m=0$. But, as used by Sommerfeld, m refers to the initial amount of angular momentum, in the case of emission, and as Kratzer has shown, the line $m=0$ is then not the missing line, but the first line of the positive branch. Hence the true moment of inertia (designated J in this article) is to be obtained directly from the value of $\Delta\nu$ corresponding to the first line to the red of the missing line (designated here as $m=+1$) while the value of $\Delta\nu$ corresponding to the missing line itself gives J', the moment of inertia of the molecule in an "energized" condition. Kratzer himself used an analytic method for obtaining the correct value of $\Delta\nu$, for the halogen acids, but an equally accurate value can be obtained readily from a simple graph of $\Delta\nu$ and m, using the value of $\Delta\nu$ corresponding to $m=+1$.

similarity of absorption and emission spectra, and to express all results, for either emission or absorption, in terms of the absorption formula.

EXPERIMENTAL DATA AND RESULTS

The λ 3883 CN band consists actually of five overlapping bands, each formed by one complete series. The positive branches of each of the five series (called the A, B, C, D, and E series) return on themselves, and thus cause the five heads of this band. Each series is a doublet series, with variable doublet separation (Δn), but corresponding terms ($\pm m$) in the two branches have the same value of Δn, another criterion for the division into branches. The value of Δn is zero for $m=0$. From here Δn increases to about 0.037 A at $m=-28$ (the first resolvable doublet) and to a maximum of about 0.075 A at $m=-75$. It then diminishes toward zero, the last resolvable doublet being $m=-126$. It is probable that the entire $\Delta n:m$ curve is symmetrical with respect to $m=75$, and can be considered as practically linear in the region of the unresolved doublets. This latter assumption was checked by the measured width of the unresolved doublets, after correcting for variation due to intensity.

Because of the blending of lines of the various series, the strong A series presents the best material for study. It has, however, been possible to secure enough data on the other series to indicate quite definitely that the results herein presented for the A series apply to each of the other series. The A series has its origin ($m=0$) at λ 3875.198, this being the theoretical position of the missing line ($m'=-28$) of the A_1 "singlet" series.[1] The positive branch proceeds toward the λ 3883 head as a series of apparently single (A_1) lines, but returns as a series of doublets (A_2). Actually, as noted, the A_1 lines are increasing in width and from analogy with the negative branch, the first resolvable doublet should be just at the λ 3883 head ($m=+28$). It is this apparent discrepancy in the two portions of the positive branch which so long

[1] If, beginning at the λ 3883 head, the lines of the A_1 series are labeled by negative values of m', while the adjoining A_2 doublet to the red is given by an equal positive value, then $m=m'+28$, always. The missing B_1 line is $m'=-29$. Hence the transformation for the B series is $m=m'+29$, while for the C series it is $m=m'+30$.

hindered a true realization of the facts. The details concerning this and other matters connected with the frequency relations in the λ 3883 band are reserved for a separate paper.

A quantitative curve of the intensity distribution of the negative branch of the A series (consisting entirely of A_1 lines) has been obtained from the series of exposures (all on the same plate) taken with a 13 ampere 170 volt carbon arc at the University of Wisconsin. The usable exposures vary from 40 minutes to 15 seconds in length. The results were obtained by finding lines of equal photographic blackening, assuming the "Reciprocity Law" (It = constant, for equal blackening). This law is known to be only approximate, and in even the modified Schwarzschild formula IT^p = constant, it appears that p can have a considerable range of values, depending on many different circumstances.[1] However, under the conditions employed by the author, it is believed that the error over the entire intensity-curve will be less than possibly 50 per cent, while for the restricted portion of the curve which is of chief value, the error should be quite negligible.

It was found that by using on the prints only lines of rather low intensity, the error of a single comparison was about 10 per cent, and this could be greatly reduced by averaging many independent comparisons. The prints were cut up, and the lines to be compared brought into actual contact. One of the inherent difficulties is due to the fact that resolved doublets have frequently to be compared with unresolved doublets. This was partly overcome by making prints on medium contrast (Azo) paper, so that none of the doublets were resolved. On maximum contrast Azo (used for both prints and enlargements) the definition is equal to that on the original negatives, while with proper exposure, the intensity variations are far more evident to the eye.

In this work it is essential that there be used only lines quite free from all other lines. While it is frequently evident that lines are "blends," it is not so evident in many other cases, and certain knowledge on this point results only from a complete

[1] See Odencrants, *Zeitschrift für wissenschaftliche Photographie*, **16**, 111, 125, 189, 1916, for complete review of work to date, and Helmick, *Physical Review*, **17**, 135, 1921, for the latest results.

analysis of all of the series of the band, i.e., a computation of the "theoretical" position of every line of every series. The author, as a result of his work on this band (*loc. cit.*) has such data for all of the stronger series, and the only series or portions of series still not analyzed are of such low intensity that they can scarcely produce any appreciable error. This data, most of which is still unpublished, has been of invaluable aid in this entire investigation. A very few theoretically "free" lines show irregular intensity, but in most cases this is evidently due to the effect of extraneous lines (line spectrum of carbon?) of which there are quite a number scattered throughout the band.

The variation of intensity throughout the observed A series lines is about seven hundred fold. The Kemble formula does not fit the intensity-curve obtained. Thus if the theoretical and experimental curves are made to coincide at the point of maximum intensity ($m = -30.4$) the experimental value at the last known line at $m = -137$ (4.5 times 30.4) is 5.5 times too large, and this evidently exceeds the possible experimental errors. As far as the chance experimental errors of the blackening comparisons are concerned, it is believed that the curve is accurate to about 10 per cent, except for very small values of m (0–5) where the error may be much greater. Other possible sources of error are (1) change of sensitiveness with wave-length of the plate, (2) uneven illumination of the printing paper, (3) uneven development of the plate or paper. It is thought that none of these causes separately would produce errors much in excess of 10 per cent. It is therefore believed that the foregoing discrepancy is due to the various approximations inherent in the use of the Kemble formula for the distribution of intensity, i.e., the neglect of the first two factors in equation (1), and the assumption of constant J.

It is convenient to have a purely empirical equation for the experimental curve, and such an expression is found to be

$$I = c_1 m^{\frac{3}{2}} e^{-c_2 m^{\frac{3}{2}}}. \tag{3}$$

The corresponding Kemble formula is

$$I = c_1 m e^{-c_2 m^2} \tag{4}$$

Equation (3) fits the experimental curve to within about 10 per cent from $m=5$ to 137, taking $m_{max.}=30.4$.

The only previous quantitative work on the distribution of intensity in this band is that of Haferkamp,[1] where, however, only the region λ 3883 to λ 3865 was studied. The quantitative portion of this work was based purely on the equality of the three lines of certain A_1A_2 triplets. At these points ($m'=15$ and 35, according to Haferkamp) the A_1 and A_2 series are taken to be of equal intensity. Actually, however, since the A_1 line is an unresolved doublet, the A_2 term is twice as strong as the A_1. In addition Haferkamp finds an irregularity at $m=-4$. This results from his misinterpretation of the A_2 perturbation at that point. The lines comprising the perturbations are, I believe, regular as regards intensity, and irregular only as regards position. The net result is that Haferkamp's intensity-curves are more or less irregular, and are not symmetrical for the two branches, a fact which has caused a real difficulty in the application of theory.[2]

Now while it is not possible to obtain directly a complete intensity-curve for the positive branch of the A series, enough intensity comparisons of the lines of the two branches can be made to make possible the conclusion that within the rather narrow limits of experimental error the distribution of intensity in the two branches is probably exactly the same. This result seems to the author of considerable importance, for it shows that these CN bands are entirely similar to the infra-red bands in intensity distribution, and thus furnishes a foundation for the use of the quantum theory of band spectra in the quantitative determination of temperature. But while the two branches seem to have intensity-curves of the same shape, the actual intensity of the positive branch averages some 10 to 20 per cent greater than that of the negative. The difference appears to be independent of the wave-length separation of the two lines compared (amounting to 3 to 85 A), and so can scarcely be due to a possible change with wave-length of the sensitiveness of the photographic plate.

[1] *Zeitschrift für wissenschaftliche Photographie*, **9**, 19, 1910.

[2] See Eucken, *Jahrbuch d. Radioaktivität und Elektronik*, **16**, 397, 1920.

Equation (1) indicates why the distribution of intensity in the two branches should be at least very closely the same. For in the case of absorption each pair of corresponding lines ($\pm m$), have the same a_m term; theory indicates that the middle factor should have the same value for the lines of a pair (i.e., the transitions $m \to m-1$ and $m \to m+1$ are equally probable for absorption); and the first factor differs at the most by 2 per cent (85 A) for the two lines of a pair.

While the experimental results thus indicate that Kemble's formula is only approximate, as applied to the complete intensity distribution in the λ 3883 band, it seems to the author probable from theoretical considerations that equation (2) giving the m_{max} should be more nearly correct. Let us first see if this equation checks in the case of infra-red bands. E. v. Bahr,[1] using the classical theory, showed that between 17° and 1430° C. the frequency separation of the maxima of the two branches was directly proportional to $\sqrt{T}$, as predicted. On quantum theory it is not the frequency separation, but the separation in terms of number of lines that should vary with $\sqrt{T}$, i.e., $m_{max} = k\sqrt{T}$. If the lines were equally spaced, as they are approximately, the two viewpoints would coincide. In the work quoted the results to be expected by the two theories differ by less than the experimental error, so that the quantum theory is equally well substantiated.

The best infra-red data is that by Colby and Meyer (*loc. cit.*) for the fundamental HCl band, at room temperature, and at a temperature described as "the tube was heated to a glow which could just be seen in a darkened room." I take this to be about 500° C. It is convenient to express formula (2) in terms of the limiting $\Delta\nu$ for $m = +1$. ($\Delta\nu_0 = h/4\pi^2 J$.) One thus obtains

$$T = 1.431\, \Delta\nu_0 m_{max}^2 \qquad (5),$$

where $\Delta\nu_0$ is expressed in frequency units. Using the accurate value of $\Delta\nu_0$ (21.30) computed by Kratzer[2] (*loc. cit.*), for $T = 300°$ absolute $m_{max} = 3.13$, for 773°, $m_{max} = 5.03$. Colby and Meyer's

[1] *Verhandlungen der deutschen physikalischen Gesellschaft*, **15**, 731, 1913.

[2] This value is from Imes's data. Colby and Meyer's data yield the same value, from the $\Delta\nu$:m graph.

curves (*op. cit.*, p. 301) indicate that these values are correct, as nearly as can be ascertained.

Let us turn again to the λ 3883 band. The outstanding feature of King's furnace spectrograms is the redistribution of intensity in each branch of the various series, as compared to the arc exposures.[1] Due presumably to the lower temperature of the furnace, the m_{max} has a smaller value, and starting at $m=0$ each branch increases very rapidly in intensity. There is a correspondingly rapid falling off of intensity for the larger values of m. Thus in my 13 ampere arc the lines $m=4$ and 83 are approximately equal, while in the 2700° furnace the equality is between $m=4$ and 52. In the 13 ampere arc $m=52$ is 4.5 times as strong as $m=83$. But again theory is verified by the fact that the redistribution of energy is the same for each branch. Thus self-reversal starts in the 2700° furnace at $m=61$ and $1\frac{1}{2}$, in each branch; and in the 2500° spectra at 51 and 3, in each branch, as nearly as can be judged. A direct comparison of $m=\pm 1$, ± 2, and ± 3 seems to indicate a small difference in absolute intensity of the two branches, but probably not as great as that in the case of the arc spectra. The ± 61 and ± 51 equalities would indicate no difference, but this criterion is not a very sensitive one.

The experimental difficulties in determining intensities are much greater, in the case of the furnace spectra, due to the self-reversal of the stronger lines. It has, however, been possible to obtain m_{max} directly by measurements of the width (a) of the self-reversed portion of the lines, since this width varies rapidly with intensity. King's furnace plates contain an arc comparison (arc of about 4 amperes, 170 volts). The width (e) of the emission line was subtracted from a. This eliminates the variation in a due to changing doublet separation. (In the region used the doublets are still unresolved.) The (a–e) computed was then corrected for the variation in e due to intensity, the amount of this correction being determined by a separate study of the width of single arc emission lines. The resulting corrected values of (a–e) gave a fairly smooth symmetrical curve, with a definite maximum. In the case of arc spectra, a direct determination (method 1) of m_{max}

[1] See King, *op. cit.*, Plate XX.

is possible, as well as an indirect method (method 2) from pairs of lines of observed equal intensity, using formula (3). The direct comparison is of course virtually the comparison of pairs of lines of equal intensity, but close to m_{max} so that the arithmetic mean may be used.

One of the first facts noted in this investigation was that King's 4 ampere arc did not show the same intensity distribution as my 13 ampere arc. Thus consider the A_1A_2 "triplet" $m'=33$ (the first to the red of the second head). In the 4 ampere arc all three lines are of practically equal intensity, as shown on King's published enlargement. In the 13 ampere arc the A_1 line is decidedly weaker than the two A_2 lines. As far as can be ascertained, equation (3) applies equally well to the 4 ampere arc. Nothing can be said as to the furnace spectra, since the data are too fragmentary. Likewise no quantative results seem possible, for the 2300° C. furnace plate, due to the weakness of the lines.

The experimental values of m_{max} are as follows:[1]

		Method 1	Method 2
13 amp. arc, m_{max}	=	30 ± 1	30.4
4 amp. arc, m_{max}	=	28 ± 1	28.2
2700° furnace m_{max}	=	23 ± 2	
2500° furnace m_{max}	=	23 ± 2	

Further material on this band is furnished by its appearance in active nitrogen.[2] Although the published photograph is one with rather small dispersion, I estimate m_{max} at 9 ± 2. The A series at this point is however contaminated by the B series, and a more accurate value is obtainable from the λ 4216 band, since here the A series is not thus contaminated. It has, moreover, practically the same $\Delta\nu_0$ and therefore its m_{max} should be the same. The result is $8\pm1\frac{1}{2}$. The peculiar appearance of these bands, in active nitrogen, is in fact due solely to the radical redistribution of intensity in each branch of the various series. Thus

[1] Since unconscious bias can easily enter into such determinations, it seems only fair to state that all of the quantitative results here presented were obtained before their quantitative theoretical significance was fully realized, and no change of any kind in the data has since been made. The value of 22 given for the furnace spectra in the August report was a preliminary one, based on a few uncorrected $(a-\epsilon)$ readings.

[2] Strutt and Fowler, *Proceedings Royal Society* (A), **86**, 1911. Plate opposite p. 117.

in the neighborhood of $m=0$, the usual appearance of the band, under low dispersion, is due almost solely to the A_2 lines, the A_1 lines being very faint. But in active nitrogen, the A_2 series is practically missing, due to the rapid decrease in intensity for higher values of m. These photographs also indicate most beautifully the similar redistribution in the other series of the CN bands. The relative intensity of the A, B, C, and D series, in active nitrogen and in King's furnace plates, seems to be changed but little, if at all.

The value of $\Delta\nu_0$ for the A series of the λ 3883 band[1] is 3.704. Putting this value in equation (5) we have

$$T=5.30\,m^2_{\max}. \tag{6}$$

From

$$m_{\max}=8\pm 1\tfrac{1}{2}, \quad T=70^\circ \text{ C.}\pm 150^\circ.$$

Strutt considers the temperature in active nitrogen about 100° C. E. P. Lewis thinks this estimate too high. The temperature must be above room temperature but probably very little above. Using the reverse process for the furnace spectrograms, we have

$$T=2700^\circ \text{ C. gives } m_{\max}=23.7$$
$$T=2500^\circ \text{ C. gives } m_{\max}=22.9$$

The experimental values were 23 ± 2 in each case. It therefor seems to the author that there is fairly conclusive evidence to the effect that over a considerable range of temperature, formula (2) is capable of giving quantitative results. This formula admittedly involves a number of real approximations, but the errors thus introduced seem to be smaller than the necessary experimental errors involved in determining $m_{\max}$.

It further seems that formula (2) should yield at least very approximate results in the case of higher unknown temperatures, in the case both of absorption and of emission spectra. It will be noted that in the previous data, the HCl band is in absorption, the active nitrogen in emission, and the furnace spectra mostly self-reversed and so a combination of emission and absorption. In postulating a similar intensity distribution for emission and

[1] Heurlinger's value of c_1 ($=\Delta\nu$ for $m=0$) is 3.84. The value of $\Delta\nu$ for $m=+1$ is obtained from $c_1-2c_2=3.84-2\times 0.068$.

absorption spectra at the same temperature, all that is assumed is that two lines of equal intensity in absorption will be of equal intensity in emission, and that the line having the greatest absorption coefficient will also show the greatest emitted intensity. The actual photometric curve of an absorption spectrum will of course depend on the depth and density of the absorbing layer. The same statement is true, to a lesser degree, for emission, as is indicated by the phenomena of self-reversal.

King's absorption plate of the λ 3883 band yields information on this subject. The plate was taken with the plug at 3500° C., and Dr. King estimates the effective temperature of the vapor as in the neighborhood of 3000° C. An accurate comparison of this plate with the furnace spectrograms is difficult, due as usual to the self-reversal in the latter, but the results are as follows. The absorption intensity relations are definitely different from those of the 2500° C. furnace plate, and as nearly as can be ascertained, the same as those of the 2700° C. plate. The difference, if any, is such as to indicate a higher temperature. I, therefore, estimate its temperature as 2700–2900° C. From the known plug temperature, this value cannot be very greatly in error.

Formula (2) can now be applied in two cases to the determination of unknown temperatures, (1) the CN vapors of the electric arc, (2) the CN vapors in the sun's reversing layer.

From the data for m_{max} already given (method 1), we have for the 13 ampere arc $T = 4500° \text{ C} \pm 300°$, and for the 4 ampere arc $T = 3900° \text{ C} \pm 300°$. It is known that the hottest portion of the arc crater is about 4000° C., but it seems very probable that the vapors are still hotter, since it is the impact of the electrons and ions of these vapors which causes the crater temperature.[1] The author is aware that there is considerable uncertainty as to just what is meant by "temperature" in a case like this, where there is no thermal equilibrium. The same problem enters into many questions concerning solar and stellar temperature. The term "effective temperature" is therefore used, leaving open the question of its relation to temperature, as measured by more ordinary methods.

[1] See Marx, *Handbuch der Radiologie*, **4**, 378–390, for summary of work on this subject.

In the case of the solar spectrum of the λ 3883 band, it is impossible to obtain directly the value of m_{max}. The only available criterion is a comparison of relative intensities of adjacent lines in the solar and in the arc spectrograms. The best lines for this purpose are the "triplet" groups in the neighborhood of $m=0$. For the A_1 lines quickly rise in intensity, going in each direction from this point, and presently equal in intensity the adjacent A_2 lines. The point where this equality occurs varies with the temperature. The available data are even more fragmentary than is indicated in Rowland's table,[1] for a detailed comparison of intensities, using enlargements of St. John's solar plate, indicates that many CN lines marked as free by Rowland are actually blends.[2] The only really available triplet groups seem to be those for which $m'=16$, 18, 19, and 26. Of these $m'=19$ (λ 3879.851, λ 3879.796, and λ 3879.716, on Rowland's scale) furnishes the most sensitive criterion. For this group, the furnace spectra and King's absorption plate show A_1 much stronger than the two A_2 lines. King's 4 ampere arc shows the three lines equal. In my 13 ampere arc A_1 is slightly weaker than A_2. In the sun the three lines seem equal. If there is any difference, A_1 is the stronger. Hence the solar temperature is in the neighborhood of the effective temperature of the 4 ampere arc (4000° C.). A study has been made also of a large number of "free" solar CN lines, and the results are in at least qualitative agreement with this conclusion. A quantitative study does not seem possible.

Grebe and Bachem's micro-photometric curves[3] yield similar results.[4] It should be noted, in this connection, that the curves on page 419 are mislabeled. The actual A_1 lines marked by dots in both arc and sun are $m'=25$, 23, and 18, and not 26, 24, and 19 as the printed wave-lengths indicate. In these curves the red is to the right, except in the solar λ 3879.716 curve. The scale is 2 cm = 1 A.

[1] *Astrophysical Journal*, **1**, 29, 1895.

[2] My first estimate of solar temperature (4700° C.) was partly due to the accidental use of such lines.

[3] *Zeitschrift für Physik*, **2**, 419, 1920.

[4] Grebe and Bachem's λ 3873.504 curves seem to furnish the best material for this test. But this triplet group ($m'=31$) is not usable in the sun, and the published "solar" curve has seemingly no relation to the actual solar spectrum at this point.

The results show that Grebe and Bachem's arc is about the same temperature as my 13 ampere arc, while their solar curves indicate a lower temperature than their arc, but possibly not as low as that of the 4 ampere arc. This temperature of about 4000° C., according to Dr. St. John,[1] is lower than anticipated, but I feel that it is probably correct to about 500° C.

The only other method for such determinations is that of Saha[2] but this method requires a knowledge of the pressure, so that the temperature determinations may be in error by large amounts. It would seem as though the present method, with increasing theoretical knowledge, should be capable of fairly accurate quantitative results. Meanwhile many possible investigations immediately suggest themselves, such as a determination of the relative temperature in sun-spots and in other portions of the solar atmosphere, and the variation of effective arc temperatures with varying physical conditions. In this connection it is to be remembered that in addition to the CN bands, others such as the λ 5165 Swan band[3] are already known to exhibit similar variations with temperature, and may therefore be used in such work.

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[1] Personal communication.

[2] *Zeitschrift für Physik*, **6**, 40, 1921, and contained references.

[3] See King (*loc. cit.*); Konen, *Das Leuchten der Gase und Dampfe*, p. 317; and Heurlinger, Dissertation, pp. 46–55.

THE ARC SPECTRA OF GALLIUM AND INDIUM

By H. S. UHLER and J. W. TANCH

ABSTRACT

Arc spectra of gallium and indium, λ 4511 to λ 2170.—To identify the lines, spectrograms obtained with the pure elements and with various compounds and mixtures were compared; these were made with a short focus concave grating spectrograph and with a large quartz prism spectrograph. *For the accurate determination of the wave-lengths,* spectrograms were made with a 21-foot concave grating, special precautions being taken to minimize pole effects, to avoid mechanical displacements and to obtain the lines sharp and separated from close lines of other elements. The wave-lengths for air, accurate in most cases to a few thousandths of an angstrom, and the vacuum wave-numbers are given for 23 gallium lines and for 34 indium lines. All the gallium lines and 28 of the indium lines were found to belong to the *two subordinate series,* for each of which from 5 to 8 terms were identified. Preliminary values of the *constants of the Ritz-Paschen formula* for these series were calculated for both elements, but the agreement between observed and calculated wave-numbers is not satisfactory. The Mogendorff-Hicks formula was tested for gallium with even worse results. Four indium lines, previously supposed to be single, were resolved into from three to six components each.

Absorption spectrum of indium in a furnace.—Specially designed furnaces were used but only λλ 4511 and 4102 were obtained.

INTRODUCTION

The present paper largely fulfils the promise made in the following quotation: "As soon as possible the wave-lengths of the gallium lines will be accurately determined by means of a 21-foot grating so that the constants of the series formula may be calculated independently of the analogous data of aluminium, thallium, indium, etc."[1]

The necessity for the investigation is sufficiently stated by H. Kayser[2] in the sentence: "Alles in Allem ist das Spectrum des Ga noch so gut wie unbekannt." The data for indium given in the fifth and sixth volumes of the same classic show that spectroscopic work on this element is also highly desirable.

The bibliography of the subject may be dispensed with by merely referring to Kayser's *Handbuch* and to a more recent paper by F. Paschen and K. Meissner.[3]

[1] Philip E. Browning and Horace S. Uhler, *American Journal of Science,* **41,** 351, 1916; see also, *ibid.,* **42,** 389, 1916.

[2] *Handbuch der Spectroscopie,* **5,** 461, 1910.

[3] *Annalen der Physik,* **43,** 1223, 1914.

The experimental work was begun before the year 1916 and it was closed in August 1919, but the attempted calculations and preparation for publication were inordinately delayed, first by the war and later by the temporary incapacity of the senior author.

The investigation involved two problems, the first of which consisted in vaporizing carefully purified metals in the arc in order to identify all of the lines radiated respectively by gallium and by indium, and to obtain the approximate wave-lengths of these lines. The second problem involved a more detailed study of the discrete lines and the more accurate determination of the wave-lengths of all the lines, except the very faintest ones.

APPARATUS, MATERIAL, AND METHOD

For the identification of the lines most of the spectrograms were obtained with two instruments: (*a*) a grating spectrograph[1] of one meter radius of curvature designed by the senior author, and (*b*) a large quartz prism spectrograph of the Littrow type made by A. Hilger and owned by the American Brass Company in Waterbury, Connecticut. Most of the films used in the grating apparatus were made either by the Wratten & Wainwright Company or to special order—through the courtesy of Dr. C. E. K. Mees—by the Eastman Company. The plates employed in the prism spectrograph were Wratten and Wainwright backed, red-sensitive plates and also special Schumann plates made and presented by Dr. I. C. Gardner. Unfortunately the plates last mentioned did not bring out any more lines in the ultra-violet than were recorded on the Wratten plates. Since the Schumann plates were of excellent quality, the negative result obtained with them indicated that the lines of high order were not radiated with observable intensity under the experimental conditions of the arcs. The metals were prepared and purified by the senior author, and they were vaporized upon positive electrodes either of Acheson graphite or of copper. This part of the work should be thoroughly reliable, as a large number of spectrograms were also taken with the materials which Browning obtained while studying the chemical separation of gallium and indium.

[1] *American Journal of Science*, **41**, 353, 1916.

For the more accurate determination of the character and wave-lengths of the stronger lines the large grating spectrograph of the Sloane Laboratory was used. The grating was ruled by J. A. Anderson; it has a radius of curvature of about 640 cm (21 ft.) with 590 lines per mm (15,000 per inch). The mounting is according to Rowland's plan and it is characterized by great rigidity. The foundation upon which the whole apparatus rests is of solid masonry and it is situated in the sub-basement of the building. The grating-room was specially designed to maintain constant temperature, a condition that is admirably fulfilled. The spectral lines between λ 4511 and λ 2932 were obtained by using both the second and the third orders, while the range between λ 2932 and λ 2300 was obtained only in the second order. The linear dispersions in the second and third orders are about 1.32 A per mm and 0.87 A per mm, respectively. The grating was very carefully tested to ascertain whether it would admit of the use of the "method of coincidences." This test was necessary because the ultra-violet lines of the third order were determined by interpolation between the standard iron lines of the second order. It was found that the grating fulfilled the requirements well within the limits of experimental error. This result was not unexpected, since it is in full accord with the investigation of Miss J. T. Howell.[1]

In several cases (e.g., Ga λ 2874) the lines under investigation sensibly coincided with iron lines and hence necessitated the use of a semaphore. The base upon which this screen was mounted was screwed to the rigid foundation of the spectrograph, very close to the incidence side of the camera. In order to avoid displacing the plate during exposure, the semaphore was raised or lowered by means of cords passing through the partition between the arc-room and the grating-room.

The plates used with the large grating were Cramer Isochromatic Instantaneous and Seed 26 and 30. Jewell's simple hydrochinone developer was invariably used with these plates.

In order to minimize the pole effect of the iron arc and to comply with the then recognized regulations for international standards, a modified form of the Pfund arc was employed.[2] This arc had a

[1] *Astrophysical Journal*, **39**, 230, 1914.

[2] *Ibid.*, **42**, 231, 1915, and **46**, 150, 1917.

length between 8 mm and 12 mm with a direct current of 5 amperes or less fed by 110 volt mains. Under these conditions it burned steadily as long as desired without restriking. A length of about 1.25 mm at the central portion of the arc was used as the source of illumination of the slit, the light from the remaining portions being screened off by a metal diaphragm which was perforated by a rectangular hole 1.25 mm high and much wider horizontally. This diaphragm was placed about 7 mm from the flame and it was rigidly attached to the base of the lamp. The upper, or negative, electrode was a rod of common carbon, while the positive electrode consisted of pure electrolytic iron kindly supplied by the United States Bureau of Standards. All the other specimens of iron tested contained manganese as an impurity, several of the lines of which nearly coincided with certain gallium and indium lines.

The secondary interferometer standards published by the Bureau of Standards[1] were used for the region between λ 4528 and λ 3233, while for wave-lengths shorter than λ 3233 the earlier data given by Fabry and Buisson were employed.

The original source of the gallium and indium was a leady residue which contained these metals as impurities to the amount of about 0.5 per cent and 2.0 per cent, respectively. This material was usually shaved into small fragments with an ice-plane and then tamped into a deep hole that had been drilled in the axis of a rod of Acheson graphite. When the lines radiated from the positive electrode thus formed were too widely reversed, the leady residue was diluted by stirring little pieces of it in ordinary liquefied lead. After the mixture or solution became homogeneous it was poured into cored carbon or graphite rods. In cases where the lines under investigation were obscured by lead lines, indium hydroxide or indium oxide, containing gallium as an impurity, was used in the cores. In other cases, lead oxide, reduced from lead nitrate which had been obtained as a by-product in the chemical work, was used to advantage. The upper electrode was invariably a rod of common carbon.

The exposure time was usually 5 min. or 10 min., but sometimes an hour or more. In some cases, half the exposure to the iron arc

[1] *Bulletin No. 274*, 1916.

was given, then the full exposure to the material in question, and, finally, the second half of the iron exposure. Check plates were always taken with the iron lines impressed simultaneously with the lines of the rare elements.

RESULTS

Gallium. Experimental.—The numerical data are collected in Table I. The numbers in the first column are merely for reference. The series notation in the second column should be obvious. The third column contains the wave-lengths in air given by Exner and Haschek. The next column presents the new wave-lengths in air obtained by the present authors. The reduction from the fourth column (15° C.) to the fifth (*vacuo*) was based on the formula given on page 722 of *Bulletin No. 327* of the United States Bureau of Standards.

TABLE I

No.	Series	λ in Air		Wave-Number $\nu_0 = \lambda_0^{-1} \times 10^8$	$\Delta\nu_0$	Int.
		E. and H.	U. and T.			
1.....	S_1 (2)	4172.06	4172.048	23962.31	826.30	10
2.....	S_2 (2)	4033.03	4032.975	24788.61	.....	10
3.....	D_1' (3)	2944.18	2944.175	33955.47	826.16	6
4.....	D_1 (3)	2943.66	2943.639	33961.66	.....	6
5.....	D_2 (3)	2874.24	2874.240	34781.63	.....	6
6.....	S_1 (3)	2719.66	2719.664	36758.39	826.23	3
7.....	S_2 (3)	2659.84	2659.873	37584.62	.....	3
8.....	D_1' (4)	.....	2500.714	39976.53	826.14	3
9.....	D_1 (4)	2500.18	2500.187	39984.96	.....	7
10.....	D_2 (4)	2450.10	2450.078	40802.67	.....	6
11.....	S_1 (4)	.....	2418.699	41331.96	825.67	2
12.....	S_2 (4)	.....	2371.325	42157.63	.....	2
13.....	D_1' (5)	.....	2338.596	42747.57	827.12	1
14.....	D_1 (5)	.....	2338.293	42752.47	.....	2
15.....	S_1 (5)	.....	2297.869	43505.12	826.15	1
16.....	D_2 (5)	.....	2294.202	43574.69	.....	1
17.....	$_1D_1'$ (6)	.....	2259.227	44249.16	826.24	1
18.....	S_2 (5)	.....	2255.034	44331.27	.....	1
19.....	S_1 (6)	.....	2236.103	44706.68	823.30	1
20.....	D_2 (6)	.....	2218.039	45075.40	.....	1
21.....	$_1D_1'$ (7)	.....	2211.753	45198.83	828.	1
22.....	S_2 (6)	.....	2195.665	45529.98	.....	1
23.....	D_2 (7)	.....	2171.9	46027.	.....	1

The differences $\Delta\nu_0$ are printed in the same horizontal line as the other data for the less refrangible of the two components

involved. This scheme avoids overlapping of braces and confusion where the subordinate series get out of step.

As is often the case, the column of intensities means but little more than is contained in the following verbal statements. The less refrangible component of each doublet is somewhat stronger than the more refrangible one. The satellites are very much weaker than either of the main components of the same doublet. In any one series the intensities of the lines decrease very rapidly as the order (m) of the line increases. When approximately pure gallium is burned in air all of the lines are widely reversed. For all the strong lines the reversal was suppressed when the metal was caused to radiate as a minute impurity in some mixture. The wave-lengths obtained with the very fine unreversed lines agreed, within the limits of experimental error, with the wave-lengths observed by setting upon the axis of the widely reversed lines.

The lines numbered 1 and 5 received special attention because of their closeness respectively to the iron lines at λ 4172.128 and λ 2874.176. Lines 1 to 5 inclusive were measured on third-order plates, while all of the lines were determined in the second order. When a line was measured in both orders, the difference between the results for each individual line was never greater than 0.004 A, and usually less. Each line was always measured four times in one direction, then the plate was reversed and the line measured four times in the other direction.

Theoretical.—In attempting to represent the series lines of gallium by an empirical equation we began by using the Ritz formula in the following form

$$\nu_m = A - \frac{N}{[m+a+b(A-\nu_m)]^2}, \qquad (1)$$

because it has been so advantageously employed by Paschen and others. In terms of the frequences ν_m, ν_n, ν_r of any three lines of the same single series the above formula easily leads to the condition

$$(\nu_r-\nu_n)\sqrt{\frac{N}{A-\nu_m}}+(\nu_m-\nu_r)\sqrt{\frac{N}{A-\nu_n}}+(\nu_n-\nu_m)\sqrt{\frac{N}{A-\nu_r}} + (r-n)\nu_m+(m-r)\nu_n+(n-m)\nu_r=0, \qquad (2)$$

from which A may be determined by the method of "trial and error." An inspection of formula (2) shows that A is not at all

dependent upon the numerical value given to m, say. This follows from the fact that m, n, and r enter equation (2) only as binomial differences. For this reason, and quite regardless of the theoretical considerations of Bohr, Sommerfeld, and others, we have chosen to define m for the first line of a given series as the smallest positive multiple of 1/2 that will make a numerically less than 1/2. When A has been determined from relation (2), formula (1) only gives the value of $m+a$ as a sum, but not of m and a separately. These remarks account for the numbers in the parentheses of the column headed "Series" in Table I.

Using R. T. Birge's latest value $N=109677.7$, Dr. E. Klein and the senior author obtained the following data from the wave-numbers of the first three lines of each single series of gallium. For the satellites of the first subordinate series $m=3$, $A'=47491.90$, $a'=-0.2126354$, $b'=+4.366673\times10^{-6}$, and $\lambda_\infty=2105.622$. For the most refrangible components of this series $m=3$, $A=48320.76$, $a=-0.2138487$, $b=+4.434379\times10^{-6}$, and $\lambda_\infty=2069.504$. For the companion of the satellite of the same series $m=3$, $A=47489.91$, $a=-0.2076978$, $b=+4.067868\times10^{-6}$, and $\lambda_\infty=2105.710$. For the least refrangible components of the second subordinate series $m=2$, $A=47553.53$, $a=+0.2138570$, $b=-2.445062\times10^{-6}$, and $\lambda_\infty=2102.893$. Finally, for the most refrangible components of the second subordinate series $m=2$, $A=48378.55$, $a=+0.2141350$, $b=-2.454515\times10^{-6}$, and $\lambda_\infty=2067.032$. These results are unsatisfactory, for the convergence wave-numbers of the least refrangible components of the two subordinate series differ by 61.63 or 2.73 A. The most refrangible components failed to converge to the same place by about the same amount, namely 57.79 or 2.47 A. Formula (1) was considered disproportionately complicated to apply the method of least squares to all of the lines of a single series. However, this method was applied to the Mogendorff-Hicks formula

$$\nu_m=A-\frac{N}{(m+a+b/m)^2}. \qquad (3)$$

The equation analogous to relation (2), as derived from equation (3), shows that the value of A depends upon the number assigned to m. By actual trial it was found that the convergence wave-numbers for the subordinate series did not agree even as well as

when formula (1) was employed. For the least refrangible components of the second subordinate series, the values of the constants in equation (3) were found, by the method of least squares, to be: $A = 47555.879$, $a = 1.2183276$, $b = -0.06226206$, and $\lambda_\infty = 2102.789$. m was chosen equal to unity for the first line. For the wave-numbers the differences "observed" minus "calculated" were $+0.041$, -0.552, $+0.824$, $+1.063$, and -1.375 or, in terms of the angstrom, -0.007, $+0.041$, -0.051, -0.056, and $+0.069$, respectively. These numbers are written in the order of decreasing wave-length of the lines. The conclusion reached by us is that the number of lines is too small and the perturbations of the lines with respect to a non-inflectional curve are too large to justify pursuing the series constants further.

No attempt was made to determine the wave-lengths of the red and green lines of the principal series of gallium for the following reasons. These lines appeared only on the spectrograms taken with the meter grating and with the large quartz spectrograph when nearly pure metallic gallium was used. They were not recorded with the oxide (which forms very promptly in the arc in air) or with the leady residue, or with any of the compounds or mixtures which contained gallium as an impurity. Whenever obtained, these lines were extremely broad and unsymmetrical. Consequently it was not feasible to make exposures with such valuable material with the large grating as mounted. Moreover, the investigation of the red region was in full sway at the Bureau of Standards, where all of the conditions are most favorable for obtaining the wave-lengths in question. The only earlier, fairly accurate, measurements of the principal series doublets seem to be those of Exner and Haschek which are quoted by Paschen and Meissner.[1]

Indium. Experimental.—Most of the investigations of the radiations of indium have been confined to the spark spectrum, with which we are not directly concerned in the present paper. The lines of the principal series have been very carefully determined by Paschen and Meissner.[2] The results of our study of the arc spectrum in air are given in Table II, the notation

[1] *Annalen der Physik*, **43**, 1223, 1914. [2] *Ibid.*

being the same as in the preceding table for gallium. Lines 1 to 22 inclusive were obtained with the large grating, while the remaining ones were recorded only with quartz spectrographs. Line number 10 was close to the iron lines at λ 2753.668 and

TABLE II

No.	Series	λ in Air		Wave-Number $\nu_0 = \frac{10^8}{\lambda_a}$ cm^{-1}	$\Delta\nu$	Int.
		E. and H.	U. and T.			
1.....	S_1 (2.5)	4511.38	4511.310	22160.05	2212.83	10
2.....	S_2 (2.5)	4101.80	4101.764	24372.88	.....	10
3.....	D'_1 (3)	3258.54	3258.565	30679.34	2212.85	4
4.....	D_1 (3)	3256.08	3256.089	30703.17	.....	8
5.....	D_2 (3)	3039.34	3039.356	32892.19	.....	8
6.....	.....	2957.02	2957.012_4	33808.33	2212.51	6
7.....	S_1 (3.5)	2932.64	2932.633	34089.03	2212.49	5
8.....	.....	2836.90	2836.911_6	35239.04	.....	8
9.....	.....	2775.36	2775.355_3	36020.84	.....	4
10.....	S_2 (3.5)	2753.89	2753.889	36301.52	.....	4
11.....	D'_1 (4)	2713.91	2713.932	36835.87	2212.53	3
12.....	D_1 (4)	2710.28	2710.264	36885.49	.....	5
13.....	S_1 (4.5)	2601.81	2601.756	38424.13	2212.73	2
14.....	D_2 (4)	2560.15	2560.157	39048.40	.....	4
15.....	D'_1 (5)	.....	2522.854	39625.34	2212.90	3
16.....	D_1 (5)	2521.32	2521.371	39649.04	.....	6
17.....	S_1 (5.5)	2468.12	2468.012	40506.23	2212.21	5
18.....	S_2 (4.5)	2460.16	2460.079	40636.86	.....	3
19.....	S_1 (6.5)	.....	2399.189	41668.05	2211.9	3
20.....	D_2 (5)	.....	2389.543	41838.24	.....	3
21.....	S_2 (5.5)	2340.27	2340.191	42718.44	.....	3
22.....	.....	2306.09	2306.073_1	43350.39	.....	4
23.....	S_2 (6.5)	.....	2278.3	43879.9	.....	2
24.....	S_2 (7.5)	.....	2240.9	44611.9	.....	1
25.....	D_2 (8)	.....	2229.8	44833.5	.....	1
26.....	S_2 (8.5)	.....	2219.4	45044.9	.....	1
27.....	D_2 (9)	.....	2210.9	45236.3	.....	1
28.....	S_2 (9.5)	.....	2202.5	45389.2	.....	1
29.....	D_2 (10)	.....	2198.2	45477.9	.....	1
30.....	S_2 (10.5)	.....	2192.0	45606.0	.....	1
31.....	D_2 (11)	.....	2189.0	45669.7	.....	1
32.....	D_2 (12)	.....	2181.7	45822.9	.....	1
33.....	.....	.....	2177.7	45906.6	.....	1
34.....	.....	.....	2171.0	46039.0	.....	1

λ 2754.034, and line number 19 to the iron line at λ 2399.339, hence the semaphore screen was used in photographing these indium lines. Special interest attaches to the four lines numbered 6, 8, 9, and 22, respectively. These lines are not single, but are made up of groups of fine components the number of which is indicated by the subscript following the representative wave-

length given in Table II. Lines 6, 8, and 9 were not obtained by Kayser and Runge,[1] but they are recorded by Exner and Haschek[1] as *single* lines with the respective characterizations 2 *R*, 3 *R*, and 1. Line number 22 is also given as single by both pairs of authors. To minimize the possibility of drawing erroneous conclusions we made a special study of these four lines and found that (*a*) each group displayed the same characteristics whether the exposure

TABLE III

Reference No.	λ	Reference No.	λ
6	2957.094	9	2775.408
	2957.047		2775.355
	2956.994		2775.292
	2956.735		
8	2837.039	22	2306.147
	2836.998		2306.082
	2836.957		2306.029
	2836.910		
	2836.856		
	2836.793		

had been long or short, the lines faint or intense, (*b*) the character of the lines was independent of the material containing the indium—whether it had been the leady residue or its products, indium oxide or hydroxide, or lead oxide, and (*c*) the lines invariably appeared when indium was present in the arc. Line number 8 suggested a small portion of a band spectrum. It could not be interpreted as a triplet, each of whose components was reversed, because all of the six components were sharp and symmetrical with no suggestion of winglike structure. The four lines are probably "combination lines," since they do not fit into the series directly. The difference in wave-number for lines 6 and 9 (when treated as unresolved) is 2212.51, which is the characteristic value for the series lines of indium. The wave-lengths of the resolved components are given in Table III.

In Table II the lines numbered 24 to 34 were always accompanied by so much intermediate radiation as to make them appear as absorption lines in a continuous background, rather than as ordinary emission lines with reversed axes. Nevertheless the serious

[1] Kayser's *Handbuch*, **5**, 584.

attempts made by Dr. E. Klein and the senior author to obtain the absorption spectrum of indium vapor, when heated in specially designed furnaces, failed to give anything more than the two strongest lines numbered 1 and 2.

Theoretical.—The time-consuming difficulties which presented themselves when the attempt was made to determine the various constants for gallium have caused us to defer a similar numerical investigation for indium until some later occasion. Moreover, as stated earlier, the presentation in published form of the experimental data has already been delayed far too long. Accordingly the following tentative constants must suffice.

Taking $m = 2.5, 3.5, \ldots\ldots$ for the first three components of the second subordinate series it was found that, for formula (1),

$$A_1 = 44449.3$$
$$A_2 = 46662.0$$
$$a_1 = a_2 = -0.215623$$
$$b_1 = b_2 = -2.96938 \times 10^{-6}.$$

Assuming the same values for A_1 and A_2, and letting $m = 3, 4, \ldots\ldots$ the remaining parameters for the satellite and the most refrangible component of the first subordinate series have the approximate values

$$a_1 = a_2 = -0.237740$$
$$b_1 = b_2 = 4.35554 \times 10^{-6}.$$

In conclusion the authors desire to make public acknowledgment of their debt of gratitude to Dr. I. C. Gardner for his Schumann plates, to Mr. W. H. Bassett, technical assistant and metallurgist of the American Brass Company, for placing at the disposal of the senior author all of the spectroscopic facilities of the research laboratory, and to Mr. C. H. Davis for his untiring and invaluable aid in obtaining the spectrograms with the large quartz prism apparatus.

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FIRST ATTEMPT AT A THEORY OF THE ARRANGEMENT AND MOTION OF THE SIDEREAL SYSTEM[1]

By J. C. KAPTEYN[2]

ABSTRACT

First attempt at a general theory of the distribution of masses, forces, and velocities in the stellar system.—(1) *Distribution of stars.* Observations are fairly well represented, at least up to galactic lat. 70°, if we *assume that the equidensity surfaces are similar ellipsoids of revolution*, with axial ratio 5.1, and this enables us to compute quite readily (2) *the gravitational acceleration at various points due to such a system*, by summing up the effects of each of ten ellipsoidal shells, in terms of the acceleration due to the average star at a distance of a parsec. The *total number of stars* is taken as 47.4×10^9. (3) *Random and rotational velocities.* The nature of the equidensity surfaces is such that the stellar system cannot be in a steady state unless there is a general rotational motion around the galactic polar axis, in addition to a random motion analogous to the thermal agitation of a gas. In the neighborhood of the axis, however, there is no rotation, and the behavior is assumed to be like that of a gas at uniform temperature, but with a gravitational acceleration ($G\eta$) decreasing with the distance ρ. Therefore the density Δ is assumed to obey the barometric law: $G\eta = -\bar{u}^2(\delta\Delta/\delta\rho)/\Delta$; and taking the mean random velocity $\bar{u}$ as 10.3 km/sec., the author finds that (4) *the mean mass of the stars* decreases from 2.2 (sun = 1) for shell II to 1.4 for shell X (the outer shell), the average being close to 1.6, which is the value independently found for the average mass of both components of visual binaries. In the galactic plane the resultant acceleration—gravitational minus centrifugal—is again put equal to $-\bar{u}^2(\delta\Delta/\delta\rho)/\Delta$, $\bar{u}$ is taken to be constant and the average mass is assumed to decrease from shell to shell as in the direction of the pole. The angular velocities then come out such as to make the linear rotational velocities about constant and equal to 19.5 km/sec. beyond the third shell. If now we suppose that part of the stars are rotating one way and part the other, the relative velocity being 39 km/sec., we have a quantitative explanation of the phenomenon of star-streaming, where the relative velocity is also in the plane of the Milky Way and about 40 km/sec. It is incidentally suggested that when the theory is perfected it may be possible to determine *the amount of dark matter* from its gravitational effect. (5) The *chief defects of the theory* are: That the equidensity surfaces assumed do not agree with the actual surfaces, which tend to become spherical for the shorter distances; that the *position of the center of the system* is not the sun, as assumed, but is probably located at a point some 650 parsecs away in the direction galactic long. 77°, lat. −3°; that the average mass of the stars was assumed to be the same in all shells in deriving the formula for the variation of $G\eta$ with ρ on the basis of which the variation of average mass from shell to shell and the constancy of the rotational velocity were derived—hence either the assumption or the conclusions are wrong; and that no distinction has been made between stars of different types.

1. *Equidensity surfaces supposed to be similar ellipsoids.*—In *Mount Wilson Contribution* No. 188[3] a provisional derivation was given of the star-density in the stellar system. The question was there raised whether the inflection appearing near the pole in the

[1] *Contributions from the Mount Wilson Observatory*, No. 230.

[2] Research Associate of the Mount Wilson Observatory.

[3] *Astrophysical Journal*, **52**, 23, 1920.

equidensity surfaces for small densities is real or not. I have since found that these inflections can be avoided without doing very serious violence to the results of observation. If this is done, the equidensity surfaces become approximately ellipsoids, and not only that, but the data can be represented without exceeding the possible limits of observation error, by assuming the equidensity surfaces to be *concentric, similar revolution ellipsoids, similarly situated.*

2. *Elements of the ellipsoids.*—Taking as unit of star-density that in the neighborhood of the sun, the adopted axes of the ellipsoids, which will be referred to as ellipsoids I, II, ... X and which

TABLE I

EQUIDENSITY ELLIPSOIDS

Ellipsoid	Log Δ	*A*	*B*	*B/A*
		parsecs	parsecs	
I	9.80−10	118	602	5.102
II	9.60	198	1010	5.102
III	9.40	296	1510	5.102
IV	9.20	413	2106	5.102
V	9.00	553	2820	5.102
VI	8.80	717	3656	5.102
VII	8.60	902	4600	5.102
VIII	8.40	1114	5675	5.102
IX	8.20	1365	6960	5.102
X	8.00	1660	8465	5.102

correspond to the values (Δ being the density) $\log \Delta + 10 = 9.8, 9.6, \ldots 8.0$, are as shown in Table I. The *A*-axis is directed toward the galactic Pole, the *B*-axis lies in the plane of the Milky Way.

For the Milky Way and for the direction toward the Pole this table yields densities which are fairly well represented, for $\rho > 150$ parsecs, by the formulae,

$$\log \Delta = -2.135 + 2.368 \log \rho - 0.593 (\log \rho)^2 \quad \text{(M.W.)}, \qquad (1)$$

$$\log \Delta = -5.356 + 4.890 \log \rho - 1.200 (\log \rho)^2 \quad \text{(Pole)}, \qquad (1a)$$

A section of the equidensity-ellipsoids through the sun (which has been assumed to be the center of the system) at right angles to the plane of the Milky Way is shown in Figure 1.

The agreement of the densities furnished by Table I with those of *Contribution* No. 188 is fairly good for all galactic latitudes up to 65° or 70°. For still higher latitudes it may perhaps still be called tolerable. At least the deviations hardly exceed what would be produced by an error of 0.1 mag. in the photometric scale for these regions.

In the present paper I have substituted these ellipsoids for the surfaces derived directly from observation in *Contribution* No. 188, *not* because I think they are nearer the truth, but simply because they are so enormously more convenient for further computation.

My aim in the present paper is simply to get hold of some approximate information about the real structure and motion of the system, and quantitative accuracy has been considered of secondary

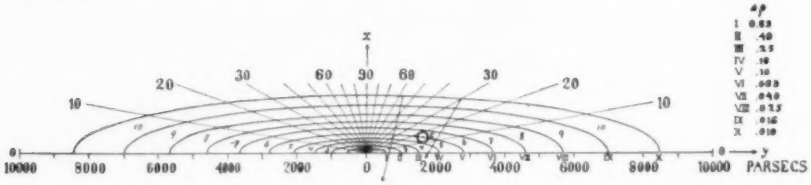

FIG. 1

importance as long as we may hope that the main features are not affected. I trust that this hope will not be disappointed, notwithstanding the many defects—defects that will be duly pointed out—which still attach to the present treatment.

3. *Advantage of the adoption of the ellipsoids.*—The form of the equidensity surfaces thus adopted has the advantage that it calls attention to the possibility of determining with some precision the gravitational attraction of the *whole* of the stellar system on any point inside ellipsoid X, while at the same time it renders the computation of that attraction a relatively easy matter.

In another paper[1] van Rhijn and I have tried to show that, as soon as we possess good counts of stars for each interval of magnitude down to apparent magnitude 17 (visual), we shall know with some tolerable approximation the density of the whole region covered by Figure 1, that is, of the whole extent of the stellar system for which the density exceeds one-hundredth of that in the neighborhood of the sun.

[1] *Mt. Wilson Contr.*, No. 229; *Astrophysical Journal*, **55**, 242, 1922.

In the near future such counts will be available. They will be furnished by the Mount Wilson "Catalogue of the Selected Areas" (from $\delta = -15°$ to $\delta = +90°$), the discussion of which is in the hands of Seares. A few provisional counts make it probable that this work will in the main confirm the elements used for Table I and Figure 1. I will assume, therefore, that even now the densities are sufficiently well known for the whole of ellipsoid X.

The advantage just alluded to is a consequence of the well-known property that the attraction of an ellipsoidal shell of constant density, bounded by two similar and similarly situated ellipsoids, on an internal point is *zero*. For it is evident by this property that, if in all that part of the system which lies outside ellipsoid X—for which part accurate data are still wanting—the arrangement in similar ellipsoids also holds, the attraction of this outside domain on a point inside ellipsoid X would be *zero*. And as the distribution of density inside ellipsoid X is known, the possibility of computing the attraction of the total system on a point inside of X becomes evident. If on the contrary the same arrangement does not hold outside ellipsoid X, it still seems highly probable a priori that any change in the form of the equidensity surfaces must be gradual, that is, the equidensity surfaces in the neighborhood of X will diverge little from similar ellipsoids, and the greater changes will begin to appear only at more considerable distances. For the consecutive shells, therefore, the attraction on an internal point will begin by being very small, both on account of the near approach to similarity of these shells and their small density and greater distance from the attracted point. For more distant shells the first circumstance will probably diminish in importance with increasing distance, while, on the contrary, the second becomes more and more important. On the whole, therefore, the attraction of all of that part of the system which lies outside X will be small, and its neglect will presumably not prevent us from obtaining fairly exact ideas about the total forces.

4. *Computation of the gravitational forces.*—In ellipsoid I, which for brevity I will call shell 1, and in each of the shells 2, 3, ... 10, between the surfaces of ellipsoids I, II, ... X, the density varies between limiting values which are in the ratio of 1 to 1.585.

In what follows I will assume for each shell a constant average density.

The computation of the gravitational forces has been carried through for:

(1) 10 points in galactic latitude 0° lying in the surfaces of ellipsoids I, II, ... X; these points have been designated by I, 0°; II, 0°; . . . . X, 0°.

(2) 5 points in galactic latitude 30°, situated on the surfaces II, IV, VI, VIII, X; denoted by II, 30°; IV, 30°; . . . . X, 30°.

(3) Similarly for the 5 points II, 57°.1; IV, 57°.1; . . . . X, 57°.1. (57°.1 = average latitude between 40° and 90°.)

(4) The 10 points I, 90°; II, 90°; . . . . X, 90°.

As a unit of attraction I have used the attraction on each other of two stars of average mass separated by a distance of 1 parsec.

I first computed the attraction of the full ellipsoids I, II, ... X on the points specified above, on the supposition that they are of a constant density such that every cubic parsec contains a single star. The formulae for this computation are given in the Appendix.

The attraction of the full ellipsoids having been found, simple subtraction gives the attraction of the separate shells 1, 2, ... 10, all supposed to have the density corresponding to one star per cubic parsec. The actual attraction of the shells was obtained by multiplying these results by the number of stars per cubic parsec contained in each shell. For the average densities, expressed in terms of the density in the neighborhood of the sun, I adopted the values corresponding to the logarithms 9.9, 9.7, 9.5, ... 8.1, each minus 10, multiplied by 0.0451, which according to *Contribution* No. 188 (12) is the number of stars per cubic parsec near the sun; this gives the numbers in Table II.

TABLE II

AVERAGE NUMBER OF STARS PER CUBIC PARSEC

Shell	No. Stars	Shell	No. Stars
1	0.0358	6	0.00358
2	.0226	7	.00226
3	.0143	8	.00143
4	.00900	9	.000900
5	0.00568	10	0.000568

Having found the separate attractions, the components of the total attractions parallel to the axes can at once be determined by noting that the attraction of any shell on an internal point is *zero*, and further, by neglecting the attraction of that part of the system outside of ellipsoid X on a point inside this ellipsoid. Instead of the components I have entered in Table III the total forces G and the angles that these forces make with the X-axis. These are of course found by the formulae

$$X=G\cos\phi \qquad Y=G\sin\phi. \tag{2}$$

In addition, I have given the values of $\phi-\psi$, ψ being the angle which the normal to the ellipsoid through the attracted point makes with the X-axis; ψ is determined by

$$\tan\psi=\frac{A^2}{B^2}\frac{\beta}{\alpha} \tag{2a}$$

The angle $\phi-\psi$ is of course the inclination of the direction of the force to the normal.

The interpretation of this table—of the first entry, for example—is: Attraction of the whole stellar system on a body in the point I, 0° is a force equal to the attraction of 33.19 stars of average mass at the distance of 1 parsec from that same body. This force makes an angle of 90° with the X-axis, and an angle of 0° with the normal to the equidensity ellipsoid through the attracted point.

5. *Analytical representation of G for galactic latitudes* 0° *and* 90°.—In trying to represent the force G by an analytical formula, I started from the consideration that, as the density is constant near the center, the attraction must be nearly proportional to the distance ρ for very small values of ρ; further, that for distances very great as compared with the dimensions of the stellar system, the attraction must be practically the same as it would be were the mass of the whole system concentrated in the center. For these latter distances, therefore, G must be proportional to $1/\rho^2$.

The following easily managed formula satisfies both conditions:

$$G=\frac{A\rho}{1+B\rho+C\rho^2+D\rho^3}. \tag{3}$$

TABLE III

Total Attractions of the Whole System

Attr. Point	Log Δ	Co-ordinates Attr. Point		G	ϕ	$\phi-\psi$	Attr. Point	Log Δ	Co-ordinates Attr. Point		G	ϕ	$\phi-\psi$
		α	β						α	β			
		parsecs	parsecs						parsecs	parsecs			
I, 0°.....	9.80−10	0	602	33.19	90°	0°	I, 90°.....	9.80−10	118	0	40.06	0°	0°
II, 0	9.60	0	1010	37.76	90	0	II, 90	9.60	198	0	54.26	0	0 0
III, 0	9.40	0	1510	37.95	90	0	III, 90	9.40	296	0	62.45	0	0 0
IV, 0	9.20	0	2106	35.71	90	0	IV, 90	9.20	413	0	65.95	0	0 0
V, 0	9.00	0	2820	32.09	90	0	V, 90	9.00	553	0	66.12	0	0 0
VI, 0	8.80	0	3657	27.71	90	0	VI, 90	8.80	717	0	64.00	0	0 0
VII, 0	8.60	0	4600	23.67	90	0	VII, 90	8.60	902	0	60.63	0	0 0
VIII, 0	8.40	0	5675	20.12	90	0	VIII, 90	8.40	1114	0	56.41	0	0 0
IX, 0	8.20	0	6960	16.59	90	0	IX, 90	8.20	1365	0	51.56	0	0 0
X, 0	8.00	0	8465	13.53	90	0	X, 90	8.00	1660	0	46.52	0	0
II, 30°.....	9.60	187	325	52.34	18° 6′	14°18′	II, 57°.1....	9.60	196	127	53.99	6°54′	5°29′
IV, 30	9.20	391	677	59.26	22 6	18 18	IV, 57.1....	9.20	410	265	64.86	8 49	7 24
VI, 30	8.80	679	1176	53.97	25 8	21 20	VI, 57.1....	8.80	711	460	62.10	10 33	9 8
VIII, 30	8.40	1055	1827	44.64	28 2	24 14	VIII, 57.1....	8.40	1105	715	53.91	12 5	10 40
X, 30	8.00	1572	2723	34.46	30 54	27 6	X, 57.1....	8.00	1647	1066	43.85	13 42	12 17

In this formula A/D evidently equals the total number of stars, N, in the stellar system.

A rough estimate of this number can be made by assuming that formula (15) of *Contribution* No. 188, viz.,

$$N_m = e^{a+bm+cm^2}, \tag{4}$$

is true for all values of m. Integration over m gives

$$N = \frac{\sqrt{\pi}}{\sqrt{-c}} e^{a-\frac{b^2}{4c}}. \tag{5}$$

For the values of a, b, c best satisfying the numbers in *Groningen Publications*, No. 27, Table V, which gives number of stars per square degree, I find the following:

Gal. Lat.	a	b	c	Number of Square Degrees
0° to ±20°	−10.564	+1.5985	−0.0276	14110
±20 to ±40	−10.539	+1.5468	−0.0288	12420
±40 to ±90	−10.815	+1.6592	−0.0414	14730

With these data, computing the total number of stars for each of the three zones, I obtained

Gal. Lat.	No. Stars
0° to ±20°	43.8×10^9
±20 to ±40	3.6×10^9
±40 to ±90	0.043×10^9

whence

$$\frac{A}{D} = N = 47.4 \times 10^9. \tag{6}$$

The remaining constants were so determined that (3) represents the values of G in Table III. I thus found

Gal. Lat. = 0°	Gal. Lat. = 90°
$A = 0.110$	$A = 0.376$
$B = 1.30 \times 10^{-3}$	$B = 1.83 \times 10^{-3}$
$C = 0.657 \times 10^{-6}$	$C = 3.40 \times 10^{-6}$
$D = 2.32 \times 10^{-12}$	$D = 7.93 \times 10^{-12}$

(7)

The representation of the tabular values is as follows:

ρ	Gal. Lat. = 0°			ρ	Gal. Lat. = 90°		
	G. Tab.	G. Form.	Tab. − Form		G. Tab.	G. Form.	Tab. − Form
500.....	30.0	30.2	−0.2	200.....	54.3	50.1	+4.2
1000.....	37.8	37.0	+0.8	500.....	66.2	68.0	−1.8
2000.....	36.0	35.0	+1.0	1000.....	58.5	60.3	−1.8
3000.....	31.7	30.2	+1.5	1500.....	49.1	49.4	−0.3
4000.....	26.0	26.0	0.0				
6000.....	19.0	19.9	−0.9				
8000.....	14.0	16.0	−2.0				

The agreement is not very good, but seems sufficient for our present purpose.

6. *Application of kinetic theory of gases.*—The results thus far obtained rest, it is true, on provisional data, which even now might be materially improved; they further depend on the supposition, not yet fully demonstrated, that, within the distances here considered, there is no appreciable extinction of light in space, but they are, nevertheless, I think, the legitimate outcome of our data.

For what follows I will now introduce some considerations borrowed from the kinetic theory of gases, the applicability of which to the stellar system might be considered doubtful. At all events I do not pretend to have demonstrated this applicability. The results which will be derived cannot lay claim to be demonstrably correct, but they seem to me to be so remarkable that, after a good deal of hesitation, I have resolved to publish them, in the hope that others, better versed in these matters, may furnish us with a more rigorous solution of the problem involved.

Even though it has been shown, in the main by unpublished investigations, that the peculiar motions[1] of the stars with some crude approximation are Maxwellian, the stellar system cannot be treated as a gas at rest; first, because of the existence of stream-

[1] Peculiar velocity is defined as the motion corrected for both the solar and stream-motion. The radial and transverse velocities agree in showing a certain excess of very large motions over the Maxwellian distribution. They are both represented satisfactorily by the sum of *two* Maxwellian distributions. A thorough separate treatment of all the spectral classes is still a great desideratum.

motion; second, because of the form of the equidensity surfaces, which is certainly different from that of the equipotential surfaces of the gravitational force.

That they are different is proved by the fact, among others, that in general the forces are not normal to these surfaces. This is evident enough without further explanation. Moreover, it is clearly brought out by Table III, where the angle with the normal reaches values of more than 27°. Further it is well known that in a gas at rest under its own attraction, the equidensity surfaces are spherical.

The system cannot therefore be in a steady state unless it has a systematic motion. Since the discovery of the star-streams it is clear that such a motion really exists and that it is parallel to the plane of the Milky Way.

It seems rational, therefore, to assume that the system has a sort of rotational motion round the X-axis (see Fig. 1) which is directed toward the pole of the galaxy. The form of the equidensity surfaces found directly in *Contribution* No. 188 as well as that now adopted. strongly indicates some such motion.

This being assumed, the stars along the axis will still have no other motion than their peculiar motions, which, as was just mentioned, are Maxwellian, at least with some approximation. I venture to assume, therefore, that the stars in the immediate neighborhood of this axis are arranged as the molecules of a gas in a quiescent atmosphere.

If:

Δ be the star-density (number of stars per cubic parsec);

u one of the components of the peculiar velocity;

η the acceleration produced by the attraction of a star of average mass at a distance of one parsec, then on the above assumption

$$\overline{u^2}\frac{\delta\Delta}{\Delta} = -G\eta\delta\rho, \tag{8}$$

$\overline{u^2}$ being the average value of u^2.

The formula is analogous to that used for barometric determinations of altitude in an atmosphere of constant temperature throughout. On the other hand, we have found empirically formulae such

as (1) and (1*a*) (see also *Contribution* No. 188, p. 13 [21]); in other words,

$$\log \Delta = -P + Q \log \rho - R(\log \rho)^3 \quad (\rho > \rho_0), \tag{9}$$

from which, by differentiation

$$\frac{\delta\Delta}{\Delta} = \frac{Q - 2R \log \rho}{\rho} \delta\rho. \tag{10}$$

Comparing the two expressions, (8) and (10), for $\delta\Delta/\Delta$

$$G\eta = -\overline{u^2}\left[\frac{Q - 2R \log \rho}{\rho}\right] \quad (\rho > \rho_0). \tag{11}$$

As the motions are supposed to be Maxwellian, the well-known formula used in the theory of least squares gives

$$\overline{u^2} = \frac{\pi}{2}(\bar{u})^2. \tag{12}$$

From observations of the radial velocities at the Lick Observatory, where no choice has been made on the basis of proper motion (*Lick Observatory Bulletin*, **6**, 126), I derived the value[1]

$$\bar{u} = 10.3 \text{ km/sec.} \tag{13}$$

or since

$$\begin{aligned} 1 \text{ kilometer} &= 3.25 \times 10^{-14} \text{ parsecs} \\ 1 \text{ parsec} &= 3.08 \times 10^{13} \text{ kilometers} \end{aligned} \tag{14}$$

I find, in the units parsec and second, here adopted,

$$\bar{u} = 3.35 \times 10^{-13} \tag{15}$$

$$\overline{u^2} = 1.763 \times 10^{-25} \tag{16}$$

so that (11) becomes

$$G\eta = -1.763 \times 10^{-25} \frac{Q - 2R \log \rho}{\rho} \quad (\rho > \rho_0) \tag{17}$$

Finally, for galactic latitude 90°, we obtain from equation (1*a*) the values:

$$Q = +4.890 \quad R = +1.200 \tag{18}$$

$$\eta = \frac{(-8.620 + 4.229 \log \rho) \times 10^{-25}}{G\rho} \quad (\rho > 150). \tag{19}$$

[1] There is a mistake in the derivation of this value. The true value is certainly somewhat lower. From considerations given below I have not deemed it necessary for the present paper to repeat the computations with an improved value of $\bar{u}$.

For small values of ρ, formula (9) does not hold. According to *Contribution* No. 188, and particularly according to *Contribution* No. 229, it represents the observations excellently for values of ρ well beyond the maximum (which in the present case lies near $\rho = 110$ parsecs). For values of ρ below the maximum the density is nearly constant. The differential-quotient $\delta\Delta/\delta\rho$ thus becomes very small and $\delta\Delta/\Delta\delta\rho$ very unreliable. In the present case it will probably be well not to rely on the formula below, say, 150 parsecs. This limit was adopted in (19).

I have computed the values of η from (19) both on the supposition that G has the values found directly in Table III and that it has the values yielded by formula (3). The former were adopted (Table IV).

TABLE IV

VALUES OF η AND $\bar{m}$

Point	ρ	η Form. (3)	η Adopted	$\bar{m}$ (Sun = 1)
	parsecs			
II, 90°	198	11.1×10^{-30}	10.2×10^{-30}	2.2
IV, 90	413	$8.9\ 10^{-30}$	$9.0\ 10^{-30}$	2.0
VI, 90	717	$7.3\ 10^{-30}$	$7.5\ 10^{-30}$	1.7
VIII, 90	1114	$6.6\ 10^{-30}$	$6.8\ 10^{-30}$	1.5
X, 90	1660	$6.5\ 10^{-30}$	$6.5\ 10^{-30}$	1.4

The quantity η is, as stated above, the acceleration per second, in parsecs, produced by the attraction of a star of average mass on a body at a distance of one parsec. The acceleration which the sun would produce, expressed in the same units, is

$$\text{Acceleration by sun} = 4.53\times10^{-30}. \qquad (20)$$

This enables us to find the average mass $\bar{m}$ of a star expressed in the mass of the sun as a unit. The values of $\bar{m}$ thus found have been inserted in the last column of Table IV.

These values agree surprisingly well with what has been found by totally different considerations. In a recent paper[1] Jackson and Furner find for visual binary stars, as the best average

$$\frac{1}{\sqrt{m_1+m_2}} = 0.855.$$

[1] *Monthly Notices*, **81**, 4, 1920.

Consequently $m_1+m_2=1.60$, which agrees with Table IV if we suppose that the combined mass of the two components and not that of a single component is comparable with the mass of a single star, and especially if we further consider that there are theoretical grounds for expecting that the average mass will decrease for increasing distance.[1]

Remark. Dark matter. It is important to note that what has here been determined is the total mass within a definite volume, divided by the number of luminous stars. I will call this mass the average effective mass of the stars. It has been possible to include the luminous stars completely owing to the assumption that at present we know the luminosity-curve over so large a part of its course that further extrapolation seems allowable.

Now suppose that in a volume of space containing l luminous stars there be dark matter with an aggregate mass equal to Kl average luminous stars; then, evidently the effective mass equals $(l+K)\times$average mass of a luminous star.

We therefore have the means of estimating the mass of dark matter in the universe. As matters stand at present it appears at once that this mass cannot be excessive. If it were otherwise, the average mass as derived from binary stars would have been very much lower than what has been found for the effective mass.

7. *Angular velocities* (ω) *in the plane of the galaxy.*—Ignoring for an instant the fact that the stars in the Milky Way cannot be systematically at rest and treating the stars near this plane in the same way as those near the axis, I am led by a formula analogous to (17) to values of η which are not quite half those given in Table IV. I suppose that the difference must be wholly due to the centrifugal force induced by the rotational motions. In fact, I assume that the average mass is the same throughout the whole system, at least for points on the same equidensity surface.

If, therefore, ρ and ρ' represent the distances from the center, of two points on the same equidensity surface, the first in the direction of the Pole, the second in the Milky Way, for which points

[1] Jeans, *Problems of Cosmogony and Stellar Dynamics* (1919), p. 239.

the total attractive forces of the system are respectively G and G', formula (11) gives

$$\text{For the Pole} \qquad G\eta = -\overline{u^2}\,\frac{Q-2R\log\rho}{\rho}, \tag{21}$$

$$\text{For the Milky Way} \quad G'\eta-\rho'\omega^2 = -\overline{u^2}\,\frac{Q'-2R'\log\rho'}{\rho'}, \tag{22}$$

where ω represents the angular velocity for the point in the Milky Way, and Q' and R' the constants of equation (9) for that same plane. The two equations determine η and ω. The equation for the latter is

$$\omega^2 = \frac{G'}{\rho'}\cdot\overline{u^2}\left[\frac{Q'-2R'\log\rho'}{G'\rho'}-\frac{Q-2R\log\rho}{G\rho}\right], \tag{23}$$

in which, for $\overline{u^2}$ see (16); for G' and G see Table V; and for Q and R see (18), and where finally, by comparing (9) and (1),

$$Q'=2.368 \text{ and } R'=0.593 \tag{24}$$

The maximum of Δ according to formula (9) lies, for the Milky Way at about $\rho=100$, for the direction toward the Pole at $\rho=110$. As has been mentioned, the formula ceases to be correct below these values. I assume, as before, that the limit of validity is $\rho=150$.

8. *Angular velocity for stars not in the galaxy.*—For the regions not in galactic latitudes 0° or 90° I determine the angular velocity by the condition that the resultant of the attractive and centrifugal forces must be at right angles to the equidensity surfaces.

In order that the system may be in the steady state, I assume that the equidensity surfaces are at the same time equipotential surfaces for the resultant of the attractive and centrifugal forces. The above condition is implied in this assumption.

Since

$$X \text{ component of resultant acceleration} = -X\eta,$$
$$Y \text{ component of resultant acceleration} = -Y\eta+\beta\omega^2,$$

and since ψ is the angle between the normal and the X-axis, the equation for ω is

$$\beta\omega^2-X\eta = -X\eta\tan\psi,$$

which becomes slightly more convenient by writing

$$X = G\cos\phi \qquad Y = G\sin\phi.$$

Whence

$$\omega^2 = \frac{G\eta}{\beta}\frac{\sin(\phi-\psi)}{\cos\psi}. \tag{25}$$

For the points in Table V, $\phi-\psi$ and ψ were taken from Table III. The computation was made with the aid of formulae (23) and (25).

TABLE V

VALUES OF ω^2

Point	ω^2	Point	ω^2	Point	ω^2
II, 0°.	0.1757 $\times 10^{-30}$	II, 30°..	0.4066 $\times 10^{-30}$	II, 57°.1..	0.4132 $\times 10^{-30}$
IV, 0 .	.0902 10^{-30}	IV, 30 ..	.2477 10^{-30}	IV, 57.1..	.2835 10
VI, 0 .	.03175 10^{-30}	VI, 30 ..	.1255 10^{-30}	VI, 57.1..	.1606 10
VIII, 0 .	.01237 10^{-30}	VIII, 30 ..	.0683 10^{-30}	VIII, 57.1..	.0949 10
X, 0 .	.00510 10^{-30}	X, 30 ..	.0376 10^{-30}	X, 57.1..	.0568 10

For these same points I have furthermore computed the linear velocities in kilometers per second. They are in Table VI.

TABLE VI

LINEAR VELOCITIES

Point	β	$\beta\omega$	Point	α	β	$\beta\omega$	Point	α	β	$\beta\omega$
		km/sec.				km/sec.				km/sec.
II, 0°..	1010	13.0	II, 30°.	187	325	6.4	II, 57°.1	196	127	2.5
IV, 0 ..	2106	19.5	IV, 30.	391	677	10.4	IV, 57.1	410	265	4.4
VI, 0 ..	3657	20.1	VI, 30.	679	1176	12.8 (13.1)	VI, 57.1	711	460	5.7 (6.3)
VIII, 0 ..	5675	19.4	VIII, 30.	1055	1827	14.7 (15.7)	VIII, 57.1	1105	715	6.8 (9.1)
X, 0 ..	8465	18.6	X, 30.	1572	2733	16.3 (17.9)	X, 57.1	1647	1066	7.8 (11.1)

9. *Explanation of star-streaming.*—According to these numbers the angular velocity is not the same for the same distance β from the axis at different distances α from the plane of the Milky Way. Further on I will explain why the present solution must necessarily be a very crude one. For this reason I am not prepared to maintain the reality of this difference. On the contrary it seems very possible that a more definite solution will finally lead to the conclusion that all the points on a cylinder around the axis of the system move with the same velocity. In fact, if we base our solution on the equi-density surfaces as really derived from the observations,[1] instead

[1] *Mt. Wilson Contr.*, No. 188; *Astrophysical Journal*, **52**, 23, 1920.

of assuming similar ellipsoids, we approach at once much closer to this state of affairs. The numbers in parentheses in Table VI give rough estimates of $\beta\omega$ based on this supposition. In the absence of a more definite solution I will confine myself mainly to points in the plane of the Milky Way, the motions of which, except for small values of β, seem to be somewhat better determined.

The most striking feature brought out by these numbers is undoubtedly the fact that at distances from the axis exceeding 2000 parsecs the linear velocity of the stars is nearly constant, the average being 19.5 km/sec.; that is, the great bulk of the stars must have a motion of 19.5 km in a direction parallel to the plane of the Milky Way. Observation has already proved that there really exists a systematic motion of the stars, that it is exactly parallel to the plane of the Milky Way, and that the motion takes place in two exactly opposite directions, the two streams having a relative velocity of about

$$40 \text{ km/per sec.} \qquad (26)$$

Since in the preceding theory the motion is introduced simply to explain certain centrifugal forces, it is at once evident that it supposes nothing about the direction in which the motion takes place. Nothing prevents us from assuming that part of the stars circulate one way, while the rest move in the opposite direction. The relative motion of the two groups will then evidently be

$$2 \times 19.5 = 39 \text{ km/sec.} \qquad (27)$$

The motion to which our theory leads, besides being in the same plane, has therefore practically the exact value which is known from observation to exist. In fact we are led in the most direct and natural way to a complete explanation of the phenomenon of star-streaming. The circumstance that observation led us to assume two rectilinear streams, whereas we here find the motion to be circular, is probably unimportant. It is of course infinitely probable that the sun must be at a certain distance from the center of the system. If we suppose it to be at the point S (see Fig. 1) then the star-streams are derived from the observed motions of stars within a volume whose dimensions are of the order of those

of the sphere around S shown in the figure. As long as the radius of this sphere is small in comparison with the distance of S from the center, the curvature of the stream-lines must be inappreciable.

When we consider that the value (27) has been obtained by a study of the arrangement of stars in space, in which the proper motions play no other part than that of a criterion of distance, while the value (26) has been obtained by a study of the motions themselves, both radial and transverse, the close agreement of the two results seems very significant. It becomes more so through the fact that both theories yield a motion exactly parallel to the plane of the Milky Way. Further, if we take into account the fact that the present theory leads to a value for the average mass of the stars which is in close accordance with what has also been found from utterly different investigations, and if we add a final point, namely, the natural explanation of the different arrangement of the stars of different spectral types, we are led irresistibly to the following conclusion:

The theory here propounded, though it may require considerable modification on account of its defectiveness both as to observational basis and mathematical treatment is probably correct in its main features.

The last point mentioned, which is open to quantitative verification, requires further investigation, but even now promises to be no less significant than the others. It is referred to again in section 14 below.

10. *Accelerations including centrifugal effect.*—If with the aid of Table V we now add the acceleration due to the centrifugal forces to that produced by the attractive force, the resultant will be in the direction of the normal. We find the results in Table VII.

TABLE VII

ACCELERATION, INCLUDING EFFECT OF CENTRIFUGAL FORCE

Point	$G\eta$	Point	$G\eta$	Point	$G\eta$	Point	$G\eta$	G_{Pole}/G_{MW}
II, 0° ..	208 × 10⁻³⁰	II, 30° ..	508 × 10⁻³⁰	II, 57°.1 .	547 × 10⁻³⁰	II, 90° ..	553 × 10⁻³⁰	2.7
IV, 0 ..	131 10⁻³⁰	IV, 30 ..	495 10⁻³⁰	IV, 57.1 .	577 10⁻³⁰	IV, 90 ..	593 10⁻³⁰	4.5
VI, 0 ..	90 10⁻³⁰	VI, 30 ..	367 10⁻³⁰	VI, 57.1 .	458 10⁻³⁰	VI, 90 ..	480 10⁻³⁰	5.3
VIII, 0 ..	65 10⁻³⁰	VIII, 30 ..	268 10⁻³⁰	VIII, 57.1 .	359 10⁻³⁰	VIII, 90 ..	384 10⁻³⁰	5.9
X, 0 ..	48 10⁻³⁰	X, 30 ..	193 10⁻³⁰	X, 57.1 .	277 10⁻³⁰	X, 90 ..	302 10⁻³⁰	6.3
ψ	90°		3°49′		1°25′		0°	

11*a*. *Velocity of escape.*—Assuming formula (3) to be valid for all distances, I find for the velocity of escape of a star near the center of the system

$$\left.\begin{array}{ll}\text{In the Milky Way} & 104 \text{ km/sec.}\\ \text{In the direction of Pole} & 98 \text{ km/sec.}\end{array}\right\} \tag{28}$$

As a matter of convenience I used for this computation the constant value

$$\eta = 7.5 \times 10^{-30} \tag{29}$$

11*b*. *Velocity compared with circular velocity.*—The velocity of a star which moves in the plane of the Milky Way in a circular orbit is determined by the formula

$$V_c^2 = G\eta\beta. \tag{30}$$

If for the point II, 0° we take $\eta = 10.2 \times 10^{-30}$, and for the others the value (29), we obtain the results in Table VIII. The linear velocities $\beta\omega$ are thus seen to be well below the critical velocity V_c.

TABLE VIII

LINEAR AND CIRCULAR VELOCITIES IN PLANE OF MILKY WAY

Point	β	G	$\beta\omega$ Table VI	V_c
			km/sec.	km/sec.
II, 0°	1010	37.76	13.0	19.2
IV, 0	2106	35.71	19.5	23.1
VI, 0	3656	27.71	20.1	26.8
VIII, 0	5675	20.12	19.4	28.5
X, 0	8465	13.53	18.6	28.5

12. *Defects of solution.*—The way in which the values ω have been determined has made the resultant of gravitational and centrifugal forces perpendicular to the equidensity surfaces. In order that the surfaces may be really equipotential there is, however, a second condition to be satisfied, viz., that for points on the same surface the total force shall be inversely as the distance of two consecutive surfaces. This condition is *not* satisfied. The difference is shown, in its most extreme form, in the last column of Table VII, which, for the points in the Milky Way and in the direction of the Pole, situated on the same ellipsoid, shows the quotient

$$\frac{\text{Force at Pole}}{\text{Force in Milky Way (including centrifugal force)}}. \tag{31}$$

If the equidensity surfaces were really accurately represented by concentric similar ellipsoids, similarly situated, as assumed thus far, this quotient in every case would be 5.1. In reality this is far from being so.

There are certain facts indicating that actually the equidensity surfaces cannot be similar ellipsoids as hitherto supposed in this article. It is a well-known fact[1] that the stars of large proper motion show no concentration toward the Milky Way, but are distributed over the various galactic latitudes with very approximate uniformity. The meaning of this can hardly be other than that the equidensity surfaces for the smaller distances are spherical. The equipotential surfaces of the gravitational and centrifugal forces, in order to coincide with the equidensity surfaces, therefore must also be spherical for vanishing distances; hence the quotient (31) must approach 1.00. Now this is just what the values of the quotient given in Table VII do. The change in these values is therefore an encouragement toward an attempt at an improved theory rather than otherwise.

13. *Position of the sun relative to the center of the system.*—Before such an attempt can be made with any hope of success it will be necessary, however, to free the data from several imperfections. Foremost among these is the imperfection in the adopted position of the sun. The data used in what precedes rest on the assumption that the sun is at the center. It seems infinitely improbable that this should be the case. Our theory, crude though it necessarily must be, paves the way for overcoming this difficulty:

First, as seen from the sun, the center of the system must lie in a plane at right angles to the true stream-motion. Adopting for the vertex of the relative motion of the two streams $\alpha = 6^h 17^m$, $\delta = +11°9$, or gal. long. $= 167°$, gal. lat. $= 0°$, we find that the center, as seen from the sun, must lie in gal. long. $77°$ or $257°$.

Second, nearly all astronomers who have dealt with the question, though from a very different point of view, agree in assuming for the center a *southerly* galactic latitude.[2]

[1] See for instance in *Verslag. Kon. Ak. v. Wetensch.*, Amsterdam, April, 1893, p. 137 (128).

[2] For instance, Struve, *Etudes d'Astronomie Stellaire*, pp. 61–62; Kapteyn, *Kon. Ak. Amsterd.*, April, 1893, p. 137; Hertzsprung, *Astronomische Nachrichten*, **196**. 207, 1914.

Third, Hertzsprung[1] from Cepheid variables finds the sun to be 38 parsecs north of the central plane of the Milky Way.

Fourth, the most difficult question probably will be that of the distance of the sun from the center. Still I think we can indicate a method which promises well.

For a first approximation, I start from the supposition, for want of a better one, that, even for the smaller distances, the *true* densities are as found in *Contribution* No. 188, which were derived on the basis of the erroneous assumption that the sun is the center. Further, for the moment, I will neglect the distance of the sun from the central plane of the Milky Way, which seems to be small.

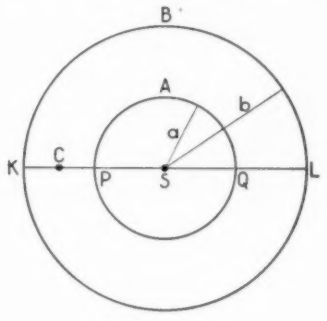

FIG. 2

In Figure 2, S represents the sun, C the center, and $CS=y$, the required distance. Let Δ_ρ represent the true star density at the distance ρ from the center. Now that we give up the erroneous supposition made in *Contribution* No. 188 and the present paper, we must consider the meaning of the densities given in Table VI of *Contribution* No. 188. What this table gives for a specified value of a (see Fig. 2) is in fact Δ', the average of the true densities at all points along the circumference PAQ around S with the radius a. This average will probably be not very different from the mean of the true densities at the points P and Q, and for the present, since our aim is only to arrive at a rough approximation, we will assume that it is exactly the case. Therefore, Δ being the true density,

$$\left.\begin{aligned}\Delta'_a &= \tfrac{1}{2}(\Delta_{y-a}+\Delta_{y+a}) \text{ if } y<a\\ \Delta'_b &= \tfrac{1}{2}(\Delta_{b-y}+\Delta_{b+y}) \text{ if } y>b\end{aligned}\right\} \qquad (32)$$

[1] *Ibid.*, p. 208.

If now, as a first approximation, we take as the true densities those obtained by formula (1) and assume in succession $y=1500$, 1000, and 500 parsecs, we find values of Δ' corresponding to different values of a as given in Table IX.

Such a table then, or something like it, would have been obtained, instead of Table VI (lat.$=0°$) of *Contribution* No. 188, if the center were actually at a distance y from the sun. Thus, if y were indeed 1500 parsecs, the considerations of *Contribution* No. 188 would have led to densities which, from distance *zero* up to distance somewhere near 1500 parsecs, would have increased. For $y=1000$ we should still have found an initial increase, and

TABLE IX

VALUES OF Δ'

a (Parsecs)	y in Parsecs		
	1500	1000	500
0	0.25	0.40	0.73
250	.26	.45	.865
500	.29	.49	.70
1000	.43	.59	.49
1500	.55	.43	.29
2000	.30	.245	.19
3000	.145	.115	.10
4000	.079	.062	.055
5000	0.044	0.039	0.035

even for $y=500$ the same would have held. In this last case, however, the increase would have been so small that it might well have been overlooked owing to the errors of observation. Since in reality the observations lead to a regular decrease of the density throughout, I think we must conclude that y cannot have as high a value as 1500 or even 1000 parsecs. As an upper limit, we may take

$$y<700 \text{ parsecs.} \tag{33}$$

We can also find a *lower* limit from the condition that the relative velocity of the two star-streams must not deviate greatly from the value of 40 km/sec. derived from observation.

In the neighborhood of the sun the linear rotatory motion of the system, that is, one-half the relative velocity of the star-streams, is $y\omega$.

If this velocity exceeded the circular velocity of a star at distance y from the center moving under the attraction of the whole system, the system certainly would not be in a steady state. Hence, on the assumption of a steady state,

$$y\omega < V_c, \tag{34}$$

or by formula (30)

$$y\omega < \sqrt{G\eta y}\,. \tag{35}$$

Assuming that inside ellipsoid II η, has everywhere the constant value 10.2×10^{-30}, we thus find, expressing results in kilometers, for

$y = 1000$ parsecs	$y\omega < 19.1$ km/sec.
700 parsecs	< 18.2 km/sec.
500 parsecs	< 16.5 km/sec.

If, therefore, we admit that $2y\omega$, the relative stream-velocity, cannot well be below 35 km/sec., we obtain as a lower limit

$$y > 600 \text{ parsecs} \tag{36}$$

The two limits (33) and (36) yield, as a first approximation,

$$y = 650 \text{ parsecs} \tag{37}$$

All these results agree fairly well in locating the center, as seen from the sun, at

Gal. long. 77°	or	$\alpha = 23^h 10^m$
Gal. lat. -3°		$\delta = +57^\circ$
Distance projected on galactic plane $= 650$ parsecs		
Distance projected at right angles to that plane $= 38$ parsecs		

(38)

The principal remaining uncertainty is perhaps that for the galactic longitude, which instead of 77° might be 257°. Personally I am strongly in favor of adopting the former value, which is in good accordance with the investigations of Herschel, Struve, and myself already quoted. My own result[1] was

$$\alpha = 0^h, \quad \delta = +42^\circ.$$

The determination (38) lays claim to no accuracy. To improve it, it will be necessary to carry through a second, and perhaps a third and fourth, approximation, for the determination of the

[1] *Verslag. Kon. Akad. Amsterdam*, January, 1893, p. 129.

lower limit of y; besides, I think we may still improve or corroborate the values of the distance of the sun from the galactic plane.[1] Altogether, the location of the center would seem to be a laborious problem rather than one of great difficulty.

There is, moreover, the possibility of a direct determination. Given sufficient data on numbers of stars and proper motions for the regions around galactic latitude 0° and longitudes 77° and 257°, there would be no difficulty in deriving separately the star-densities at different distances from the sun in these regions. If there is truth in the above theory, these densities must *increase* with the distance up to distance y, in the direction *toward* the center, whereas in the opposite direction they must decrease with increasing distance. The two solutions together must yield a rather crucial test of the whole theory.

14. *Further defects. Separate treatment of the different spectral types.*—There are several further defects in the solution of the present paper. I will enumerate those that occur to me:

a) In the investigation on which the present paper is based the average parallax, as a function of magnitude and proper motion, has been taken from *Groningen Publication*, No. 8. At the present moment we have already available, though not yet published, much improved values, especially for the very small proper motions.

b) The value of $\overline{u^2}$ is not the very best that could have been obtained.

c) The values of G in Table III have been computed on the supposition that the average mass of a star is independent of the distance, whereas later, in section 8, it was found that this is not the case.

d) In this same computation for G, it was further assumed that the attraction of the whole of the system outside ellipsoid X on an internal point is zero. Though reasons were given for admitting that this attraction is small, it may not be negligible. Since according to what precedes it is highly improbable that the

[1] Furthermore the distance of the center from the sun, as here found, is so small that, if special attention is given to the matter, it may not be hopeless to get evidence of the curvature of the stream-lines, particularly for the rather distant stars (faint stars having small proper motion).

equidensity surfaces within X, as well as without, are really similar ellipsoids, the whole computation for G will have to be revised in a more definitive treatment.

e) The present solution is for all the stars together, that is, for a very heterogeneous collection of stars.

Only the last point is of fundamental importance, and I will devote a few lines to it. Just as in the higher, very attenuated parts of an atmosphere, where the molecular encounters become relatively rare and the different gases are, as it were, sorted out, so among the stars we must expect that the different spectral classes will show different arrangements in space, owing to the different values of $\overline{u^2}$ and η (mass) peculiar to these classes. The two cases are not identical, owing to the stream-motion in the stellar system, which is not supposed to exist in the atmosphere. Still there must be analogy. So, to take an extreme example, if there exists a class of stars for which $\overline{u^2}$ is zero, these stars, according to the present theory, would be confined exclusively to the central plane of the galaxy. For evidently a star not in that plane could not have pure stream-motion; that is, it could not move in a perfect circle around the axis and perpendicular to it, because the attractive force of the system on such a point does not lie in that plane. Therefore it would necessarily have a peculiar motion.

This simple consideration makes us understand at once the otherwise astonishing fact that the Wolf-Rayet stars lie with such close approximation in the central plane of the Milky Way. In the present theory this means simply that they can have no other than pure stream-motion (possibly with some peculiar motion *in* the plane of the Milky Way[1]). We thus realize that for the several classes of spectra there must be a very intimate connection between the values of $\overline{u^2}$ and η on the one hand, and the well-known differences in concentration toward the galaxy on the other.

Such considerations show that for the totality "all stars together" the value of $\overline{u^2}$ will doubtless change with the position in space and in particular with the distance of the stars from the plane

[1] It is noteworthy that, provided the peculiar motion *in* the Milky Way is also *zero* (as it must be when the Maxwellian distribution holds for this limiting case), the parallax of these stars becomes a pure function of the linear velocity, that is, of the quotient, radial velocity divided by sin λ, λ being the angular distance, star-apex.

of the Milky Way. The present investigation, therefore, suffers from the defect that this circumstance was neglected, $\overline{u^2}$ having been taken the same for the whole of the system. I think that this shows sufficiently the absolute necessity of treating the different spectral classes separately.

For such a treatment the necessary data are not yet available, or at least not available in a satisfactory form. Fortunately, however, there exist at present instruments capable of dealing successfully with difficulties which, not so long ago, would have been insurmountable. With their aid we may hope to obtain material for several spectral classes separately, little or not at all inferior to what has already been obtained for the stars as a whole. I think that the determination of the spectral class for some 1000 stars of magnitudes 11 to 12, well distributed over the sky and for which the proper motion is either already sufficiently well known, or may be determined by taking a few additional photographic plates, will go far toward supplying us with what is so urgently wanted.

APPENDIX

Formulae for the computation of the attraction of a homogeneous revolution ellipsoid, of unit density (1 star per cubic parsec), on an exterior point.

Take the axis of revolution as X-axis and let the xy-plane contain the attracted point whose co-ordinates are α and β. Let A and B be the axes of the ellipsoid, the first coinciding with the axis of revolution, and let X and Y represent the components of the required attraction, Z being zero. Then from the well-known formulae,[1] if we put

$$\lambda^2=\left(\frac{B}{A}\right)^2-1 \qquad (h)$$

$$K=\frac{2K}{\lambda^3}\left(\frac{B}{A}\right)^2 \qquad (k)$$

$$p=\lambda\frac{A}{A'}, \qquad (l)$$

[1] I have used the formulae as given by Duhamel, *Cours de mécanique*, V, 1, pp. 321 and 318.

where, if $\alpha=0$, A' is obtained from

$$A'=\sqrt{\beta^2-A^2\lambda^2} \qquad (m)$$

and if $\beta=0$, from

$$A'=\alpha. \qquad (n)$$

For other values of α and β we may write

$$E^2=\frac{\lambda^2}{2}-\frac{\alpha^2+\beta^2}{2A^2} \qquad (o)$$

$$\eta=\lambda\frac{\alpha}{A} \qquad (p)$$

$$\left(\frac{A'}{A}\right)^2=-E+\sqrt{E^2+\eta^2} \qquad (q)$$

With these auxiliary quantities we find

$$X=-\alpha K\phi\ (p), \qquad (r)$$

$$Y=-\beta K\omega\ (p), \qquad (s)$$

in which $\phi(p)$ and $\omega(p)$, or rather their logarithms, have been tabulated in Table X by means of the formulae

$$\phi(p)=2[p-\text{arc tan } p] \qquad (t)$$

$$\omega(p)=\text{arc tan } p-\frac{p}{1+p^2} \qquad (u)$$

TABLE X

VALUES OF LOG ϕ (p) AND LOG ω (p) ACCORDING TO FORMULAE (t) AND (u)

p	Log ϕ	Log ω	p	Log ϕ	Log ω	p	Log ϕ	Log ω
0.000	$-\infty$	$-\infty$	0.05	5.920^{-10}	5.920^{-10}	0.5	8.861^{-10}	8.804^{-10}
1	0.824^{-10}	0.824^{-10}	6	6.157	6.156	0.6	9.076	8.997
2	1.727	1.727	7	6.358	6.357	0.7	9.252	9.149
3	2.255	2.255	8	6.531	6.530	0.8	9.399	9.272
4	2.630	2.630	9	6.685	6.682	0.9	9.524	9.372
5	2.921	2.921	10	6.821	6.819	1.0	9.633	9.455
6	3.158	3.158	11	6.945	6.942	1.1	9.728	9.525
7	3.359	3.359	12	7.058	7.054	1.2	9.812	9.585
8	3.533	3.533	13	7.161	7.157	1.3	9.886	9.635
9	3.687	3.687	14	7.257	7.252	1.4	9.954	9.679
10	3.824	3.824	15	7.346	7.341	1.5	0.015	9.717
11	3.948	3.948	16	7.430	7.423	1.6	0.070	9.750
12	4.061	4.061	17	7.508	7.500	1.7	0.121	9.780
13	4.166	4.166	18	7.581	7.573	1.8	0.168	9.806
14	4.262	4.262	19	7.651	7.642	1.9	0.211	9.828
15	4.352	4.352	20	7.717	7.707	2.0	0.252	9.850
16	4.436	4.436	21	7.779	7.768	2.1	0.289	9.869
17	4.515	4.515	22	7.839	7.827	2.2	0.325	9.885
18	4.590	4.590	23	7.896	7.882	2.3	0.358	9.900
19	4.660	4.660	24	7.950	7.935	2.4	0.389	9.914
20	4.727	4.727	25	8.002	7.986	2.5	0.418	9.927
21	4.790	4.790	26	8.052	8.035	2.6	0.446	9.939
22	4.851	4.851	27	8.100	8.081	2.7	0.472	9.950
23	4.909	4.909	28	8.146	8.126	2.8	0.497	9.960
24	4.964	4.964	29	8.190	8.169	2.9	0.521	9.969
25	5.018	5.017	30	8.233	8.210	3.0	0.544	9.977
26	5.069	5.068	31	8.274	8.250	3.1	0.566	9.985
27	5.118	5.118	32	8.314	8.288	3.2	0.587	9.993
28	5.165	5.165	33	8.352	8.325	3.3	0.607	0.000
29	5.211	5.211	34	8.390	8.361	3.4	0.626	0.006
30	5.255	5.255	35	8.426	8.396	3.5	0.645	0.012
31	5.298	5.297	36	8.461	8.429	3.6	0.663	0.018
32	5.339	5.339	37	8.495	8.461	3.7	0.680	0.023
33	5.379	5.379	38	8.528	8.493	3.8	0.697	0.028
34	5.418	5.418	39	8.560	8.523	3.9	0.713	0.033
35	5.456	5.455	40	8.591	8.552	4.0	0.728	0.038
36	5.492	5.492	41	8.621	8.581	4.1	0.743	0.042
37	5.528	5.528	42	8.651	8.609	4.2	0.758	0.046
38	5.563	5.563	43	8.680	8.635	4.3	0.772	0.050
39	5.597	5.596	44	8.707	8.662	4.4	0.786	0.054
40	5.630	5.629	45	8.735	8.687	4.5	0.799	0.057
41	5.662	5.661	46	8.761	8.712	4.6	0.812	0.060
42	5.693	5.693	47	8.787	8.736	4.7	0.825	0.064
43	5.724	5.723	48	8.813	8.759	4.8	0.837	0.067
44	5.754	5.753	49	8.837	8.782	4.9	0.849	0.070
45	5.783	5.782	0.50	8.861	8.804	5.0	0.861	0.072
46	5.812	5.811						
47	5.840	5.839						
48	5.867	5.866						
49	5.894	5.893						
0.050	5.920	5.920						

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A CONTINUOUS SPECTRUM FROM MERCURY VAPOR

By C. D. CHILD

ABSTRACT

Continuous spectrum of *mercury, red to ultra-violet.*—(1) *Conditions for emission in discharge tube.* A continuous spectrum identical with the fluorescence spectrum is emitted in addition to the ordinary line spectrum only when (*a*) the temperature of the vapor is above 120° C. and below 300° to 400°, depending on the pressure, when (*b*) the pressure is above 1 mm, when (*c*) the current-density is low, of the order of 10^{-4} amp/cm² or less and (*d*) when very little air or other contaminating gas is present. An influence machine or transformer is better as a source of current than an induction coil, especially one with condensers. Ionization of the vapor is not necessary for the emission of this spectrum since it was obtained with only 7 volts between the Wehnelt cathode and surrounding cylinder. (2) *In explanation* it is suggested that *the carriers of the spectrum* are molecules consisting of two or more atoms, which emit the fluorescence spectrum as a result of excitation by ultra-violet radiation λ 2536 emitted by atoms struck by electrons with energies corresponding to 4.9 volts or more.

Continuous spectrum of sodium.—Preliminary observations on heated sodium vapor in a discharge tube indicated that a continuous spectrum was present but much fainter than the line spectrum.

Ionization potential for mercury.—Some observations on the current from a heated filament to a concentric electrode as a function of voltage seem to show that the ionization potential *varies with the temperature*, reaching a minimum for about 140°, when it was 3 volts less than for 50°.

Change in chemical behavior of mercury vapor at about 120°C.—Above that temperature it *reacts with air* to form a dark compound, but not below. This reaction is associated with the appearance of the continuous spectrum.

The writer has recently observed that under certain conditions the electric discharge through mercury vapor produces a glow that is distinctly green. This shows the well-known line spectrum of mercury and also one that is apparently continuous from the red into the blue, being most prominent to the eye in the green. A brief mention was made by Warburg[1] in 1890, that he had observed a continuous spectrum when studying the electric discharge through mercury vapor. This was verified by Wiedemann and Schmidt,[2] Kalähne,[3] and Kowalski.[4] A continuous spectrum was also found by Hartley[5] when mercury vapor is excited to fluorescence by ultra-violet radiation, and was further studied by Wood[6]

[1] *Wiedemann's Annalen*, **40**, 14, 1890.

[2] *Ibid.*, **57**, 457, 1896. [3] *Ibid.*, **65**, 826, 1898.

[4] *Physikalische Zeitschrift*, **15**, 249, 1914.

[5] *Proceedings Royal Society*, A, **76**, 428, 1905.

[6] *Philosophical Magazine* (6), **18**, 240, 1909, and (6), **23**, 689, 1912.

and others. However, no extended study has been made of this spectrum when excited by an electric discharge, and the investigation described in the following paragraphs was, therefore, undertaken.

Conditions necessary for obtaining the continuous spectrum. Small current-density.—There are three conditions necessary for obtaining the continuous spectrum by an electric discharge with any considerable brightness; the first being that the current-density must be small. As the current-density is increased the line spectrum becomes more prominent and the continuous spectrum less so. Contracting the tube as in the Plücker tube has the same effect as increasing the current.

The best source of current for showing this spectrum is the Wimshurst or Toepler-Holtz machine. With the current from such a machine it is often impossible to detect any lines even when the continuous spectrum is very bright. A step-up transformer may also be used to advantage. An induction coil is not so satisfactory. It is possible to obtain the continuous spectrum with the discharge from a hot wire covered with calcium oxide, if the current is sufficiently small. Putting condensers in parallel with the induction coil has the same effect as increasing the current. In most of the experiments described in this paper the discharge was obtained from a transformer, the current being approximately 5×10^{-5} amperes per square centimeter.

Temperature and density necessary for the continuous spectrum.—The second condition necessary for obtaining the continuous spectrum is that the temperature must be above 120° C. and below a higher temperature which depends on the density of the vapor. Kowalski found that mercury vapor excited by an electrodeless discharge gave a white glow at room temperature and a green glow when heated to 140° C. This is approximately the same temperature as that which was found necessary to produce the continuous spectrum.

A third condition necessary for the continuous spectrum is that the vapor have a density approximately that of saturated vapor at 120° C. This may be shown by maintaining the lower part of a tube containing a small amount of liquid mercury at a temperature

below 120° C., while the upper part is kept above that temperature. The vapor in such a tube is less dense than that required and does not show the continuous spectrum. This will be discussed more fully in a following paragraph.

Even when the density of the vapor is kept constant the continuous spectrum becomes less prominent, if the temperature is raised too high. This was shown by a tube 25 cm long which was heated while being exhausted until all but a few minute drops of the liquid mercury were vaporized. It was then sealed off from the pump and further heated until the mercury was completely vaporized. This showed the continuous spectrum with scarcely a trace of the line spectrum. If it was further heated until the glass became slightly conducting the glow became distinctly blue with the discharge concentrated in a line in the center of the tube. The line spectrum was then prominent with but little of the continuous spectrum.

An experiment by Wood[1] showed that the same effect occurs at a higher temperature when the continuous spectrum is excited by ultra-violet light. In his experiment a quartz tube containing a very small drop of mercury was exhausted and sealed off from the pump and heated until the mercury was completely vaporized and the tube became red hot. It was found that after the mercury had all vaporized the fluorescence diminished in intensity and finally disappeared.

It is difficult to determine the exact temperature and density necessary to produce the continuous spectrum not only because any change in the spectrum due to heating the tube occurs gradually, but also because any trace of gas other than mercury hinders the production of the continuous spectrum. When a tube is sealed off from the pump and used for a time such gases are apt to occur. Lastly it is difficult to distinguish between effects due to temperature and those due to density, since it is impossible to increase the density of the vapor without increasing the temperature.

However, the arrangement shown in Figure 1 gives results of some value.

[1] *Philosophical Magazine* (6), **18**, 246, 1909.

The tube T was connected to the pump by a trap d. Observations were made with this open and with it closed. The upper parts of T and c were surrounded with an asbestos jacket which was heated by a ring burner. The lower part, including d, was immersed in an oil bath which was kept at a lower temperature. When the trap at d was open the tendency for the mercury to distil into the pump was hindered by placing a capillary tube 20 cm long and 1.25 mm in diameter at c.

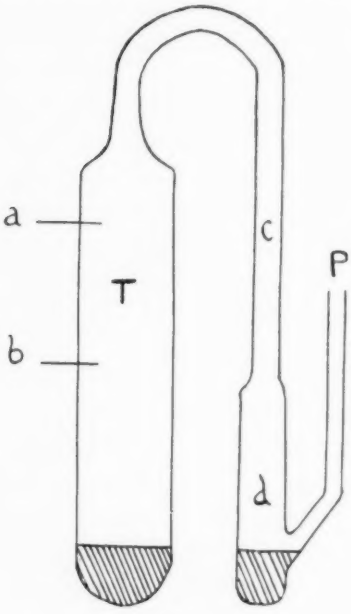


FIG. 1

The trap at d could be closed by distilling mercury over from the tube until the opening to P was closed. Both with the trap open and with it closed the tube was heated for some time before observations were taken.

It was found that both the upper and the lower parts of the tube must be at least at 130° C. in order to show the continuous spectrum prominently. Heating the upper alone was not sufficient even when the lower was but a few degrees below the required temperature. This again shows that it is not only necessary to have a required temperature but also a certain density of the vapor.

The lower part of the tube was then maintained at various temperatures above 130° C. and the upper part was heated until the continuous spectrum approximately disappeared. In this case the density of the vapor did not remain constant but diminished as the temperature of the upper part was raised. The results of such observations taken with the trap open are given in Table I but approximately the same results were obtained with the trap closed. Column 1 gives the temperature of the bath surrounding the bottom of the tube, and Column 2 gives the approximate

temperature of the air inside the jacket above which the vapor ceases to give the continuous spectrum. Above 400° C. the glass became somewhat conducting, so that no higher temperature could be taken with the apparatus at hand.

TABLE I

Column 1	Column 2
132	310
140	335
150	375
160	410
167	440

As has been stated, the values given in the second column are rough approximations, but they are sufficiently accurate to prove that the temperature at which the continuous spectrum disappears depends on the density of the vapor in the tube. This is what might be expected if the continuous spectrum depends on the presence of molecules in the vapor. According to the mass law, if both atoms and molecules are present, the greater the density of the vapor the greater the relative number of molecules at a given temperature, and the higher the temperature needed to break up all of the molecules into atoms.

If both the vapor and the liquid mercury are heated above 200° C. the discharge becomes concentrated at the center of the tube. At 280° C. with a pressure of 15 cm the discharge is almost a line, which is very bright and shows the line spectrum much more prominently than the continuous spectrum.

Effect of vaporization and condensation.—It is easier to obtain the continuous spectrum where vaporization and condensation are taking place, as may be shown by heating a tube at some part where a few drops of mercury have condensed. If a discharge is passed through the tube, the continuous spectrum is much more prominent in the region where the vaporization occurs than elsewhere.

However, neither vaporization nor condensation is essential for the production of the continuous spectrum. This was shown in the experiment already described where a tube was heated until the mercury was completely vaporized. When there was neither

vaporization nor condensation, there was still the continuous spectrum with scarcely a trace of the line spectrum.

There is, however, no question but that condensation aids in showing the continuous spectrum and also that it is the condensation rather than the vaporization that is helpful. It is somewhat difficult to prove this since newly formed vapor ordinarily expands as soon as it leaves the surface of the hot mercury and in doing so may condense enough to show the continuous spectrum. However, that the preceding statement is correct was shown by the tube illustrated in Figure 2. The electrode *a* extended into the liquid mercury. This was heated and the part of the tube at the left of *c* was kept as hot as possible. The part at the right was allowed to remain comparatively cool. The connection to the pump was left open, so that only a part of the vapor passed into the tube. Under these conditions the glow over the liquid mercury and as far as *c* was white, while that at the right of *c* was green. By having a current slightly greater than the minimum it is possible to obtain a discharge which shows no continuous spectrum at the left of *c* while it shows it very prominently at the right.

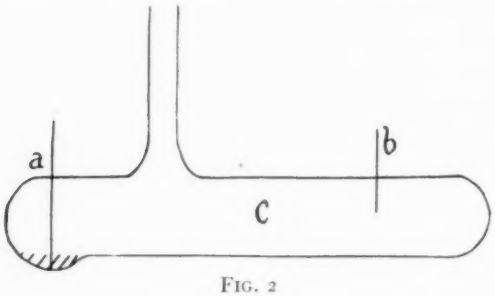


FIG. 2

Effect of other gases on the mercury vapor.—Wood[1] found that mercury would not fluoresce when mixed with other gases. The same has been found true when the continuous spectrum is produced by an electric discharge. As has been stated, a very slight amount of air changes very greatly the upper temperature at which the continuous spectrum disappeared. Again, with 2 or 3 mm of air and a small amount of mercury vapor it is impossible to obtain a continuous spectrum at any temperature. Thus in the preceding experiment, with the bath at the bottom of the tube

[1] *Astrophysical Journal*, **26**, 43, 1907; *Philosophical Magazine* (6), **18**, 244, 1909, and (6), **21**, 314, 1911.

at 140° C. and 3 mm of air present, no continuous spectrum could be obtained, whatever the temperature of the upper part of the tube. When the upper part was at 230° C. only the line λ 5461 could be detected. A somewhat similar phenomenon has been observed by Robertson[1] when exciting mercury vapor by electrodeless discharge.

Continuous spectrum not affected by the kind of electrode, kind of glass, nor purity of mercury.—The continuous spectrum is not affected by the shape of the tube nor that of the electrode, providing the current-density is not increased. It is shown equally well with platinum, iron, or mercury electrodes and in tubes made of soda, lead, or Pyrex glass. It showed equally well with mercury that had been distilled three times and with mercury containing zinc, lead, cadmium, or sodium as impurities.

Mercury was also purified by distilling it in a stream of air. It is claimed by Hulett and Minchin[2] that mercury so distilled has less than one part of zinc to 10^{10} parts of mercury and that other oxidizable metals are removed to an equal extent. Mercury thus purified showed the same spectrum as other mercury.

Differences between normal mercury vapor and that giving the continuous spectrum. Ionization of the gas not necessary for radiation.—In a number of ways the vapor which is in a condition to give the continuous spectrum behaves differently from mercury vapor at room temperature. First it may be made to give visible radiation without being ionized. This might be expected from the fact that it fluoresces under the influence of ultra-violet radiation without becoming ionized, as has been shown by Wood.[3] That such radiation may also be produced by an electric discharge without ionizing the vapor is shown by the following experiment. Current from a hot wire covered with calcium oxide was passed to a surrounding cylinder. When the mercury at the bottom of the tube was heated so as to send a stream of hot vapor through the cylinder the region above the cylinder showed a faint glow which gave the continuous spectrum even when the voltage between

[1] *Science*, **52**, 386, 1920.

[2] *Physical Review*, **21**, 388, 1905.

[3] *Philosophical Magazine* (6), **23**, 713, 1912.

the wire and the cylinder was much below that needed to produce ionization.

In the experiment as performed the wire was 4 cm long and 2 mm in diameter. The surrounding cylinder was of iron, 5 cm long and 2 cm in diameter. At approximately 7 volts there was a faint glow above the cylinder, which grew somewhat brighter as the voltage was raised, but there was no large increase in the current until a difference of potential of 12 volts was applied, when the current suddenly increased and the region between the wire and the cylinder became luminous. The change above 12 volts was no doubt due to the beginning of ionization of the vapor. This would, therefore, seem to justify the conclusion that the continuous spectrum may be produced without ionizing the vapor.

A lower ionization potential.—Again the ionization potential of the vapor giving the continuous spectrum appears to be lower than that of normal mercury vapor. This was shown by examining at different temperatures the voltage at which there occurs a large increase in the current passing from a hot wire to a surrounding cylinder without any corresponding increase in the voltage. The same tube was used for this purpose as in the preceding experiment where the discharge from a hot wire was examined. The air was pumped out by a diffusion pump until no reading could be made on a McLeod gauge that read down to .0002 mm. The tube was then heated for some time so as to drive off any residual air, the tube becoming filled with mercury vapor from mercury kept in the bottom of the tube. The current through the platinum wire was kept at 4.5 amperes.

At approximately 50° C. the critical voltage was 16 volts. When the tube was heated until the continuous spectrum appeared the critical voltage was approximately 13 volts. On further heating until the mercury at the bottom was freely boiling the critical voltage increased to 18 volts. On allowing the tube to cool this voltage again decreased to 13 volts and then increased to 16 volts.

Data taken with different temperatures of the wire and with different sizes of wire all showed a minimum critical voltage when the tube was barely hot enough to show the continuous spectrum.

A check upon this is given by the fact that when a current is sent through a tube from a Wimshurst machine, the potential difference between the electrodes is smallest when the temperature is such that the continuous spectrum begins to appear. This was shown by rough measurements taken with an electroscope.

The high voltage required when the temperature was high was no doubt due to the large density of the mercury vapor. Although impact between electrons and atoms of mercury at low voltage is elastic, a small amount of energy is lost to the electron at each impact, so that with large density much of the energy is lost and a high voltage is necessary to produce ionization.

The decrease in voltage occurring when the temperature is such that the continuous spectrum is barely visible must, of course, be explained in some other way. It might be thought that at the lower temperatures there was some impurity in the gas which caused the critical voltage to be high. This does not, however, seem probable, first, because the amount of gas other than mercury vapor which was left in the tube was very small, its pressure being less than .0002 mm. Secondly, even this small amount would be swept out by the flow of mercury vapor into the cooler parts of the apparatus.

Again the lower critical value occurring when the continuous spectrum appeared might be explained on the ground that the platinum wire became hotter as the tube was heated and that the apparent critical voltage became lower because of the higher velocity of the emitted electrons. However, the resistance of the wire was measured by a Wheatstone bridge arrangement and it was found that when the tube was heated and the current through the wire was kept constant, the resistance of the wire became slightly less, indicating that the temperature of the wire decreased instead of increasing. This, no doubt, was due to the cooling effect of the mercury vapor about the wire. As the temperature of the tube is raised the density of the vapor increases and the denser vapor is able to conduct more heat from the wire, so that its temperature is lower, although the temperature of the surrounding tube is higher.

It, therefore, seems necessary to accept the view that the lower critical voltage is due to the fact that the ionization potential of the

mercury vapor is less when the vapor is in a condition to give the continuous spectrum than when it is at room temperature. Whether this lowering is due directly to a change in the condition of the molecules which makes them more easily ionized, or to some ability which they acquire of absorbing radiation, cannot at present be stated. A more direct method of measuring the ionization potential is desirable in order to give us further knowledge of this subject.

Hot vapor active chemically.—During these experiments it was noticed that if any appreciable amount of air was left in the tube and the temperature raised sufficiently high to produce the continuous spectrum, a dark compound was formed which stuck to the sides of the glass. The substance was not formed when the discharge was passed through a tube at room temperature. By using a tube that was hotter at the bottom than at the top it was possible to observe the boundary between the region showing the continuous spectrum and that showing the line spectrum and it was found that this was also the boundary between the glass covered with the deposit and the clean glass.

Continuance of the glow.—When this discharge showed the continuous spectrum there was usually a green glow for several centimeters above the upper electrode. This could be explained as due to carriers of the continuous spectrum continuing to radiate for an appreciable time after they were excited. This continuance of the glow was examined by two different methods. One of those used has been described in a preceding article.[1] The discharge was passed from the secondary of a transformer and the time at which the current ceased was compared with the time at which the light ceased to be visible. This method indicated a continuance of the light for .002 seconds after the current ceased.

There is, however, an objection to this method owing to the possibility of the glass's becoming charged at the time the discharge passes and producing a luminous discharge after the current proper has ceased. The following arrangement obviates this difficulty to a large extent. The tube which has already been described in showing the discharge from a hot wire to a cylinder

[1] *Physical Review* (2), **9**, 1, 1917.

surrounding it was used. The current between the wire and the cylinder was made and broken by a wheel attached to the synchronous motor used in the preceding experiment. The difference in time between the breaking of the current and the time when the light ceased to be visible was apparently .008 seconds.

Since the voltage here was not more than 30 volts and the wire was so nearly surrounded by the cylinder, it seems as if the glass could not have been charged to a sufficient amount to produce any light. On the other hand the fact that the light was here observed to continue for a period longer than that found with a spark discharge which is known to be somewhat untrustworthy makes it desirable to study this phenomenon further.

Character of the spectrum.—As far as could be determined by observations with the eye and by some rather crude photographs of the spectrum the continuous spectrum here studied is the same as that given by Wood[1] but the writer has neither the apparatus nor the experience to warrant further work with this. Wood's work has shown that this spectrum is continuous from the yellow into the ultra-violet at about the wave-length λ 3000 with a minimum at λ 3600. When the glow is produced by electric means and the conditions are at their best the red is also visible.

Again we might expect the relative intensity of different parts of the spectrum to vary as the conditions of the experiment varied. Such a variation was found by Wood[2] in the case of iodine vapor when its density was changed. A difference of this sort would indeed be expected if the molecules giving the spectrum differ in size or are subject to any change which progresses as the temperature is varied.

However, there does not appear to be any such variation in the continuous spectrum from mercury vapor, as is shown by the following experiments. A discharge was passed from the bottom of a tube containing boiling mercury to an electrode 24 cm above it. It is probable that if the molecules change either in size or in condition, they do so in passing from the hot liquid to the cooler region above. However, no difference could be observed in the

[1] *Philosophical Magazine* (6), **18**, 240, 1909. Plate 7, Fig. 6.

[2] *Ibid.* (6), **27**, 531, 1914.

character of the spectrum coming from different parts of the tube. Neither did changing the intensity of the light produce any change in the relative brightness of different parts of the spectrum.

Again the color of the discharge through tubes differing in temperature and in density of vapor is the same as long as only the continuous spectrum appears. For example one tube was heated to approximately 140° C. and second to 220° C. The pressure of the vapor in the first case was 2 mm and in the second was 3 cm. When the two tubes were placed side by side and the same current sent through them no difference in color could be detected. At higher temperatures the discharge becomes concentrated in the center of the tube and the line spectrum becomes noticeable. In that case a difference of color is, of course, noticeable.

Moreover, Wood and others who have examined the fluorescence of mercury vapor when at atmospheric pressure mention the color as being green, which is the same as that given by the vapor at low pressure.

The evidence obtained from the spectrum would, therefore, indicate that the oscillating system which gives the continuous spectrum is distinctly different from that which gives the line spectrum, that the light always comes from the same oscillating system rather than from a number of different ones and that it is not affected by the frequency of the impacts between the molecules.

The carriers of the continuous spectrum.—In attempting to determine the carriers of the continuous spectrum two explanations have been considered. One is that the same vibrating system gives both the line and the continuous spectrum, the vibrations being modified when the vapor is dense by the greater number of impacts. There are two objections to this view. First, experiment shows that when the density is kept constant and the temperature of the vapor is raised the ability of the vapor to give the continuous spectrum is impaired, if not destroyed. But under these conditions there is an increase in the number and intensity of impacts per second and if the continuous spectrum is due to impacts between atoms, one would expect the continuous spectrum to become more conspicuous instead of less so.

Secondly, there are impacts at 120° C. as well as at 140° C. They are indeed more than twice as frequent in the latter case and somewhat more intense, but if the impacts affect the oscillations at one temperature, one would expect some effect at the other and yet at the lower temperature there is usually no indication of the continuous spectrum, while at the higher one it almost entirely displaces the line spectrum. Moreover, the usual change in the spectrum to be produced by increasing the density is a broadening of the lines and not a complete change of the spectrum.

It seems to the writer that a more probable explanation is that the carriers of the continuous spectrum are molecules of mercury while the atoms are the carriers of the line spectrum. The determination of the specific gravity of mercury vapor indicates that it is monatomic, but these determinations are not sufficiently accurate to make the presence of the molecules seem improbable even when the temperature is high, and it is difficult to believe that a monatomic vapor can give two such radically different spectra and behave in other ways as differently as this vapor does.

Such an assumption would explain the fact that with high temperatures or with large currents such as occur with the mercury arc the continuous spectrum does not appear, for in such cases the molecules are no doubt split up into atoms which give only the line spectrum.

This, however, does not explain the large change in the spectrum which occurs at 130° C. We are, therefore, compelled to make the further assumption that the molecules are more easily excited at high temperatures than at low. Such an assumption is not unreasonable. While energy is probably not absorbed by the internal structure of isolated atoms when the gas is heated, it probably is absorbed by molecules of two or more atoms. If so, such molecules may be more easily ionized and excited to radiate at higher temperatures. According to such an assumption, at room temperatures the atoms are the more easily excited and only the line spectrum appears, while at the higher temperatures the molecules are more easily excited and we have also the continuous spectrum. The facts can, therefore, be explained on the following assump-

tions: *First, the carrier of the continuous spectrum are molecules consisting of two or more atoms, and secondly, these molecules are more easily excited at high temperatures than at low.*

Manner of excitation.—There are at least two ways in which the vibrations giving the continuous spectrum may be excited; the molecules may be agitated by impacts of electrons on the molecules, or by the absorption of ultra-violet radiation. The impact of electrons on the atoms of mercury no doubt produces radiation of wave-length λ 2536 and in a similar way the impact on molecules may produce the continuous spectrum, but since this spectrum is certainly produced by ultra-violet radiation, as was shown by Wood, and since radiation of wave-length λ 2536 is produced by the impact of electrons on atoms as has been shown by Franck and Hertz, it seems to the writer more probable that there is first a production of the radiation of wave-length λ 2536 and that this under proper conditions causes fluorescence of the vapor which gives out the continuous spectrum.

When an electron which has passed through a difference of potential of 4.9 volts, hits a mercury atom, it gives up its energy to the atom and the atom radiates this energy as ultra-violet light. When the gas is as dense as in the cases here studied the greater part of the energy goes primarily into such radiation, for with such densities the electrons will in almost every case hit an atom after it has passed through 4.9 volts and before it has passed through 10.4 volts, which is the voltage necessary to produce ionization.

An atom which has thus been partially ionized may be hit by a second electron before it radiates its energy and complete ionization may occur, as has been pointed out by Van der Biji[1] and Compton.[2] The recombination resulting from such ionization produces the line spectrum of mercury. With larger currents the chance of an atom being hit a second time before it has radiated its energy is, of course, greater than with small currents, so that entirely aside from the increase in temperature caused by the large current the line spectrum is more apt to be prominent when the current is large.

On the other hand, if the partially ionized atom is not quickly hit, it will radiate its energy. This may pass to a second atom

[1] *Physical Review* (2), **10**, 546, 1917. [2] *Ibid.* (2), **15**, 476, 1920.

and partially ionize it, which in turn may give out this same kind of radiation. This last is the kind of fluorescence which is known as resonance radiation.[1]

If there are no molecules present, or if they are not in a condition to absorb this energy, as they apparently are not at room temperature, then all of the energy of this radiation will either aid in ionizing atoms or will be radiated from the gas as wave-length λ 2536. If, however, there are molecules of two or more atoms present which are in a condition to absorb the energy of this radiation, they fluoresce, emitting a radiation which gives the continuous spectrum.

The manner of excitation of the continuous spectrum is then conceived to be as follows: The electrons partially ionize some of the atoms. These radiate energy of wave-length λ 2536. If this radiation falls on molecules which are capable of absorbing it, the vapor fluoresces, giving out the radiation which gives the continuous spectrum.

The experiment which has been described showing that it is possible to produce the continuous spectrum without ionization confirms this explanation.

The fact that the line spectrum appears at the cathode even when the continuous spectrum appears in the positive column may be explained by assuming that in the cathode drop where the electric force is large the electrons attain sufficient velocity to completely ionize the atoms upon impact, and the resulting recombination of the positive ions with the electrons gives the line spectrum.

The condensed discharge.—As has been stated the discharge from an induction coil does not give as prominent a continuous spectrum as that from a Wimshurst machine. This is especially true, if condensers are in parallel with the tube. A tube so heated that the discharge from a Wimshurst machine will show only the continuous spectrum will scarcely show anything but the line spectrum with the spark from an induction coil.

There are two respects in which the condensed discharge differs from the uncondensed. The maximum current is greater with the

[1] *Philosophical Magazine* (6), **23**, 699, 1912.

condensed discharge, and the voltage when the spark commences is higher due to the fact that the ions have nearly all recombined since the preceding discharge. The following experiment indicates that it is the large current rather than the high voltage which causes the difference. The current from a Wimshurst machine was passed between *a* and *b* in Figure 3 when the tube was so heated as to give the continuous spectrum. A spark from an induction coil having capacity in parallel with the tube was then passed between *c* and *d*. This was found to show the line spectrum as plainly between *a* and *b* as in the other parts of the tube. The voltage between *a* and *b* could not in this case have been greater with the spark since the glow discharge would keep the gas in this region conducting while the current from the induction coil was interrupted.

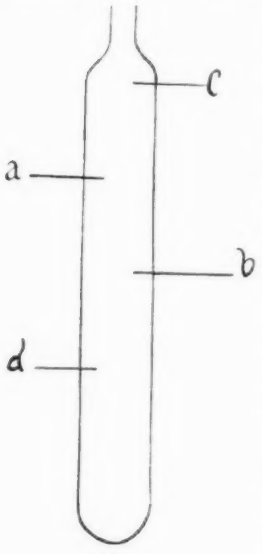


FIG. 3

Continuous spectrum from other gases.—If this explanation is correct, it would be reasonable to expect a continuous spectrum from other gases, or at least with other monatomic gases when near the point of condensation. Some observations were made on the discharge through sodium vapor when heated. There appeared to be a continuous spectrum present but it was much fainter than the line spectrum, so that no very definite description of it can be given. It is probable that it was the fluorescent spectrum of sodium vapor which has been described by Wiedemann and Schmidt[1] and others.

I desire to express my appreciation of the assistance which has been given me in the preceding work by Mr. B. K. Northrup, of Cornell University, and Professor A. E. Wood, of Colgate University.

COLGATE UNIVERSITY
September 1921

[1] *Annalen d. Physik*, **56**, 18, 1895.

FORMATION AND LIFE OF METASTABLE HELIUM

By FABIAN M. KANNENSTINE

ABSTRACT

Current-potential curves for alternating and intermittent arcs in pure helium between a Wehnelt cathode and nickel anode were obtained by use of a Braun-tube oscillograph, for pressures of from 0.06 to 2 mm. With alternating voltages, the striking potential for each half-cycle was 29 volts or more for frequencies of 200 or less, but for frequencies of 220 cycles and above, the current started at about 5 volts and soon reached a saturation value until about 25 volts caused a further increase. In all cases the breaking potential was about 5 volts. To form the intermittent arc, the voltage was periodically dropped from 36 to some lower predetermined value. If this lower potential was less than 4.6 volts, the arc was suddenly extinguished; if between 4.6 and 23.7 volts, the arc died out gradually; and if it was 25.3 volts or above, the arc was maintained. All the foregoing values include a correction of 1.1 volts added to the actual readings to bring them in agreement with accepted results.

Evidence as to the life of metastable helium.—To explain various experimental results, it has been suggested that atoms of helium, partially ionized by electronic collisions of 20.8 volts or more and hence capable of complete ionization by further impacts of 4.8 volts or more, remain in this metastable condition for an appreciable time. The foregoing results not only confirm this suggestion but also show that under the circumstances of this experiment the metastable form persists for about 0.0024 second.

INTRODUCTION

Experiments with electron impacts in helium have led to a value for the minimum ionizing potential of 25.3 volts and to two radiating potentials, one at 20.5 volts[1] and another at 21.3 volts.[2] Franck and Reiche[3] and Franck and Knipping[2] conclude that the partially ionized state formed when the atom has been excited by the impact of 20.5 volt electrons is a "metastable" state, which may persist for some time and may be even permanently stable in absolutely pure helium. This conclusion is drawn from the fact that the radiation usually observed with electron impacts at this voltage disappeared when the helium was very pure, indicating that the displaced electrons do not return to the normal state. In slightly impure helium they also observed that the radiation, as measured by

[1] F. Horton and A. C. Davies, *Philosophical Magazine*, **42**, 746, 1921; J. Franck and P. Knipping, *Physikalische Zeitschrift*, **20**, 481, 1919; K. T. Compton, *Philosophical Magazine*, **40**, 553, 1920; F. S. Goucher, *Proceedings Physical Society of London*, **33**, 13, 1920.

[2] J. Franck and P. Knipping, *loc. cit.* *Zeitschrift für Physik*, **1**, 320, 1920. F. Horton and A. C. Davies, *loc. cit.*

[3] *Zeitschrift für Physik*, **1**, 54, 1920.

its photo-electric effect, appeared to continue for a brief time after the exciting electron impacts ceased.

Experiments by Paschen,[1] which show that in pure excited helium resonance radiation may be caused by the infra-red line 10830 A, also support the conception of a metastable state. All the energy of this wave-length absorbed by the excited atoms was found to be re-emitted as radiation of this same wave-length. In terms of Bohr's model this means that the radiation is absorbed by atoms which have already absorbed the energy corresponding to the radiating potential 20.5 volts, and that the displaced electrons return to this partially ionized state only, and not to the normal state.

The semi-stable existence of partially ionized helium atoms is also useful in explaining the phenomena in low-voltage arcs in helium, especially the striking of the arc at 20.5 volts with dense electron currents.[2] The phenomena of successive impacts as discussed by Compton[3] are of much more importance when the partially ionized atoms have a relatively long life. The metastable helium should be completely ionized by electrons with a speed corresponding to a potential of $25.3 - 20.5 = 4.8$ volts. If we assume that the metastable state may be produced by the arc radiations,[4] then an arc might be maintained with dense enough electron currents down to 4.8 volts. It is of interest that in this way Compton, Olmstead, and Lillie have maintained an arc with potentials as low as 8 volts between the electrodes. Horton and Davies[5] report an unsuccessful attempt to observe the ionization of abnormal helium atoms by bombardment with electrons of 4.8 volts.

In this paper experiments with helium arcs are described which show that helium may exist temporarily in a metastable form, and that arcs may be maintained in it with 3.5 volts. An approximate determination of its life is also reported.

[1] *Annalen der Physik*, **45**, 625, 1914.

[2] K. T. Compton, E. G. Lillie, P. S. Olmstead, *Physical Review*, **16**, 282, 1920.

[3] *Physical Review*, **15**, 476, 1920.

[4] O. W. Richardson and C. B. Bazzoni, *Nature* **98**, 5, 1916; F. Horton and A. C. Davies, *op. cit.*, **42**, 752, 1921.

[5] *Ibid.*, **42**, 763, 1921.

ARCS WITH LOW-FREQUENCY VOLTAGES

The arcs were formed between a Wehnelt cathode and nickel anode in a pyrex tube containing carefully purified helium. The helium was purified by passing over heated copper oxide and charcoal immersed in liquid air. Before passing into the experimental tube the helium passed through two additional charcoal tubes and a liquid-air trap. The tube and liquid-air trap were baked out at 450° C. for several days before the helium was admitted.

In the first experiments the helium was excited by an alternating e.m.f. An electromagnetic oscillograph was used to obtain curves showing the variation of current and voltage with time. From such curves the current-voltage curve may be obtained.

A decided dissymmetry in the growth and decay of the arc current could be observed, and also an apparent continuation of the arc current down to very low voltages. To obtain greater sensitiveness and to avoid the possible distortion from the inertia of the oscillograph elements, a sensitive Braun tube oscillograph was used in most of the later experiments. The electrostatic deflecting plates of the Braun tube were connected across the anode and cathode of the helium tube, while the arc current passed through the electromagnetic deflecting coils. The electrostatic and electromagnetic deflections were at right angles, so that the figure on the screen of the Braun tube was a graph in rectangular co-ordinates of the impressed e.m.f. and the arc current. The deflecting coils were so adjusted that the thermionic current alone did not produce any observable deflection.

Figure 1 shows the kind of curve obtained with a 60-cycle e.m.f. in helium at a pressure of 2 mm. It is seen that at a certain voltage the arc strikes, and as the voltage increases the current increases also. When the voltage decreases the current decreases at the same time, but continues with considerable magnitude until the voltage reaches a value of about 4 volts, when the current ceases to flow. Careful observation showed a sudden drop to zero at this lower voltage. This continuation of the arc to very low potentials may be attributed to the persistence of a metastable form of helium in which an arc may be maintained down to about 4 volts. Two other breaks in the down curve may be observed

just below the striking potential of the helium itself. These breaks will be referred to later.

This type of curve was observed with 60 cycles at pressures down to 0.06 mm. Down to 1 mm pressure the striking potential was about 28 volts, and at the dying-out potential of about 4 volts a very decided drop of the arc current to zero was observed. As the pressure was decreased below 1 mm the striking potential

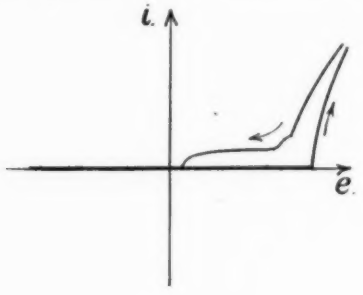


FIG. 1

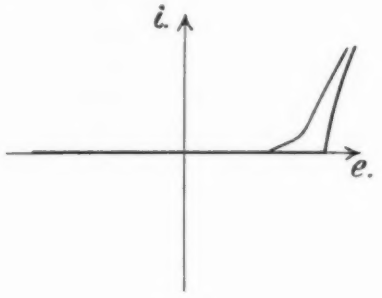


FIG. 2

increased and the dying-out potential became less certain, i.e., the drop to zero current was not so sudden as at higher pressures. At 0.06 mm the striking potential had increased to about 40 volts and the dying-out potential to about 20 volts. Figure 2 shows the type of curve obtained at 0.06 mm pressure. At all these pressures the filament temperature was approximately constant. The mean arc current varied from 25 milliamperes at 2 mm pressure to 10 milliamperes at 0.06 mm pressure.

If any impurities are present one would expect their effect to be more marked at lower pressures. Franck and Knipping have assumed that metastable helium will not return to normal helium except in the presence of impurities, so that with larger amounts of impurities we might expect the metastable helium to be destroyed in a shorter time. This is probably the interpretation of the gradual dying out of the arc before the voltage had dropped to 4 volts, as was observed when the pressure was very low. Experiments were made directly with the method discussed in the next section to see if the life was affected by impurities obtained by removing the liquid air from the charcoal for a short time till the helium had become very impure. The experiments indicated

that while the impurities suppressed the amount of the metastable helium, the part remaining had the same life as that formed in the pure helium. The increase in the striking potential as the pressure was reduced is more difficult to interpret and is probably connected with the mechanism of the arc formation.

ARCS WITH HIGH-FREQUENCY VOLTAGES

With a sufficiently high frequency the e.m.f. should pass from +4 volts to its negative maximum and back to +4 volts before the metastable helium has disappeared, and upon reaching 4 volts the arc should strike, due to the ionization of the metastable helium still present.

Accordingly various frequencies from 60 to 10,000 cycles were impressed on the arc. An audion oscillator was used as a source for these frequencies. The author wishes to acknowledge his indebtedness to Dr. J. P. Minton and Mr. V. O. Knudsen for the use of their oscillators in this work. The alternating e.m.f. impressed on the arc was obtained from the secondary of an iron core transformer having its primary winding in the plate circuit of the oscillator.

Figure 3 shows the type of figure observed above 220 cycles per second at 2 mm pressure. The arc struck at about 4 volts, and the current rapidly attained a saturation value at which the arc current continued until the voltage had reached about 25 volts, when a second sudden increase was obtained. The decreasing part of the curve was above the increasing part but fell to zero at the lower striking voltage.

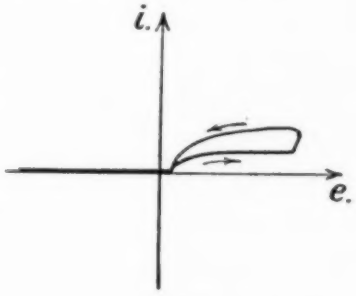


FIG. 3

With 200 cycles no appreciable arc formation at the lower voltage could be observed, indicating that the metastable helium practically disappeared in less than approximately $\frac{1}{400}$ of a second. The striking of the arc at about 4 volts with 220 cycles indicates that enough of the metastable helium was left after approximately $\frac{1}{440}$ of a second to give an observable indication of ionization.

All the figures above 220 cycles were similar, except that as the frequency was increased the striking at about 4 volts became more and more marked.

In using the oscillator as a source of alternating e.m.f. smooth curves were not always obtained. As the mean arc current was increased, by raising the electron emission, certain parts of the curve became wavy. This was particularly noticeable on the striking of the arc at about 25 volts. Instead of the arc current increasing as in Figure 1, it increased in a wavy form, indicating that the impressed voltage rose and fell during this striking period. This is provisionally attributed to self-induction in parts of the circuit or to the presence of iron in the oscillating circuit.

ARCS WITH INTERMITTENT CONSTANT VOLTAGES

In another series of experiments intermittent constant voltages from a battery were impressed between the hot cathode, *C*, and the anode, *A* (see Fig. 4). When the mercury interrupter, *H*, was open the voltage impressed on the electrodes was about 35 volts, and when the interrupter shorted the section *BD*, the voltage dropped to a value dependent upon the position of the slider *B*. In this way it was possible to impress a voltage of about 35 volts and suddenly change this voltage to a lower value. The arc current was passed through the deflecting coils, *M*, of the Braun tube. The spot was examined directly and also with a rotating mirror. Since it was rather difficult to observe the spot in the rotating mirror the following arrangement was also used. A uniform time deflection was obtained by connecting the deflecting plates to a potentiometer with a rotating contact, which was mounted on the shaft of the mercury interrupter. The electrostatic deflection was therefore proportional to the time and the

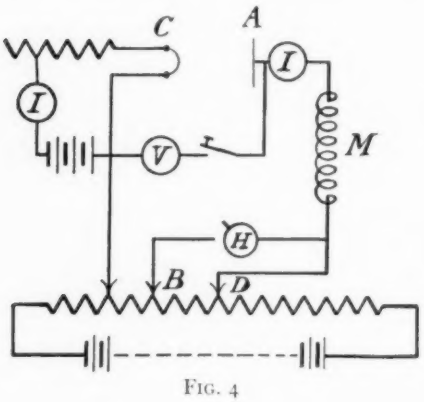


FIG. 4

magnetic deflection proportional to the current through the arc. These two deflections were at right angles. Furthermore, since the rotating contact was mounted on the interrupter shaft, the figure produced was stationary and could be carefully examined as the slider *B* was moved.

With the lower voltage below 3.5 volts and without the time deflection two spots appeared on the Braun tube screen, the upper one corresponding to the current for 35-volt excitation and the lower one for zero current. Using the time deflection, it was seen that the arc was extinguished in a time too short to be so observed, as shown at the right of Figure 5. Only a very faint vertical line appeared, since the spot moved very rapidly from the upper to the lower positions.

As the lower voltage was increased to 3.5 volts the lower line suddenly became curved as shown in Figure 6, indicating a gradual

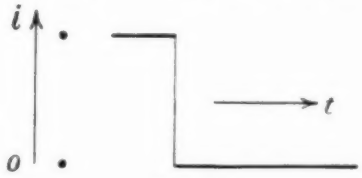


FIG. 5

FIG. 6

decrease of the current to zero value. This sudden curving of the lower line at 3.5 volts may be attributed to the presence of metastable helium produced during the 35-volt excitation. That is, the voltage dropped from 35 volts to 3.5 volts in a time short compared with the life of the metastable helium, and the arc was maintained by 3.5 volts as long as the metastable helium was present.

The dying out of the arc continued to have the same appearance until the lower voltage had been increased to 22.6 volts, when a third spot appeared slightly above that for zero current as in Figure 7. Using the time deflection, it was seen that the arc was maintained for a brief time with this voltage before dying out.

A slight further increase of the lower voltage to 23.6 volts brought out still another spot, as in Figure 8, and the time deflection showed a dying out of the current in two stages as shown. The voltages for these two breaks in the decrease correspond to

the voltages for the two breaks observed in the down curve of the alternating e.m.f. figure already described. This decrease in two stages may be connected with the fact of the existence of two radiating potentials of helium 0.8 volts apart corresponding to the first coplanor orbit (helium) and the first crossed orbit (parhelium) on Franck and Knipping's interpretation.

With a slightly increased lower voltage of 24.2 volts the arc was maintained. The two lower spots of Figure 8 disappeared, and the arc current jumped between the two values indicated by the two upper spots.

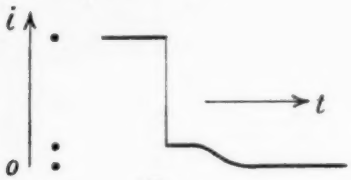


FIG. 7

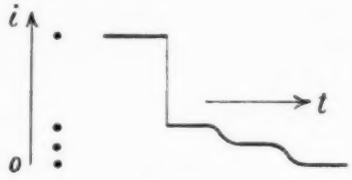
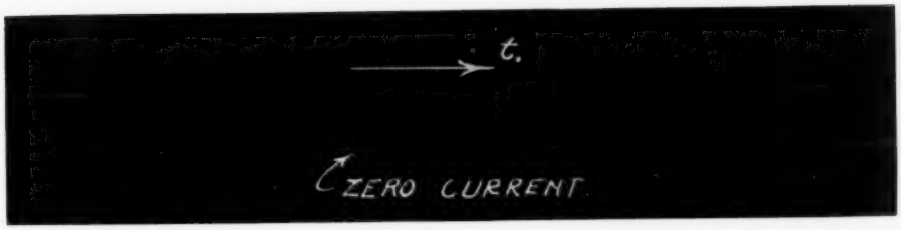


FIG. 8

These phenomena were qualitatively the same at pressures ranging from 2 mm down to 0.06 mm. The above exact determinations were all made visually with the Braun tube oscillograph, but for the purpose of illustration photographs made with the electromagnetic oscillograph show qualitatively the same phenomena, although with less sharpness in the details. Plate VI shows the various changes which occur as the lower voltage is increased from zero to 24.2 volts. These oscillograms were obtained by passing the arc current through an element of the electromagnetic oscillograph and photographing the curve while the mercury interrupter was operating. In *a* the lower voltage was below 3.5 volts, and it is seen that the arc current drops to zero in a very short time. The curving of the lower current line when the lower voltage reaches 3.5 volts is shown in *b*. The brief maintenance of the arc at a lower voltage of 22.6 volts is shown in *c*, and the decrease in two stages at a lower voltage of 23.6 is shown in *d*. The maintenance of the arc at 24.2 volts is shown in *e*.

A number of readings were taken of these voltages, and then the cathode current was reversed in order to eliminate the drop in voltage along the cathode, and another set of readings taken. The

PLATE VI



a

b

c

d

e

above values are the mean of all these values and thus are the potentials correct to approximately 0.1 volt between the anode and the center of the cathode. Corrections are necessary to allow for the initial velocity of emission of the electrons, the contact difference of potential, and the fact that some of the electrons are emitted from a more negative part of the filament than the center. As the drop of potential from the center to one end was 0.9 volt, this last correction may amount to about 0.4 volt. The other two corrections may easily amount to one volt or more according to a recent theory of K. T. Compton[1] in explanation of the results obtained by Hebb[2] on the arcing voltage in mercury vapor. It is thus permissible to apply a correction of 1.1 volts to the observed values given to make the upper voltage required to maintain the arc, 24.2, agree with the observed value of the ionization potential in helium, viz., 25.3 volts. This correction applied to the lowest potential observed for striking or maintaining the arc, 3.5 volts, gives a value of 4.6 volts in agreement within experimental error, with the theoretical ionization potential for metastable helium. Or, avoiding the question of corrections entirely, we may say that the difference between the two voltages observed, the lowest potential required to maintain an arc in helium with low electron density, and the minimum potential to maintain an arc in metastable helium is $24.2-3.5=20.7$ volts, and this value is within experimental error the minimum potential required to produce the metastable form of helium.

The author wishes to express his thanks to Professor H. G. Gale for his kind encouragement and advice during the progress of this work, and to Professor A. J. Dempster, who proposed the investigation of this particular phase of helium arcs, and without whose suggestions and assistance this work would not have been possible.

RYERSON PHYSICAL LABORATORY
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[1] American Physical Society Meeting, Washington, April 22, 1922.

[2] *Physical Review*, **16**, 375, 1920.

NOTES ON IONIZATION IN THE SOLAR ATMOSPHERE[1]

By HENRY NORRIS RUSSELL[2]

ABSTRACT

Ionization in the solar atmosphere.—The absorption of photospheric radiation by atoms in the sun's atmosphere tends to increase the degree of ionization, both directly, by shifting an electron into a position from which its removal is easier, and indirectly, when enhanced lines are absorbed, by getting the ionized atoms into states in which they are probably less likely to combine with electrons. Both these influences operate strongly in the case of barium and weakly, if at all, for sodium. This probably explains why barium is much more highly ionized in the sun than sodium, although their ionization potentials are the same. The case of lithium, which is also ionized to an abnormally high degree, cannot be explained in this way. The greater width of the lines, due to the thermal agitation of so light an atom, may help to solve the problem.

Barium.—The lines of this element which are present, or doubtful, in the solar spectrum are tabulated. All the strong lines arise from the ionized atom. A few faint lines of the neutral atom are probably present.

Rubidium.—Three enhanced lines of this metal are present, though faint, in the normal solar spectrum.

Manganese.—The ionization potential 7.6 volts agrees well with the value predicted from the behavior of its lines in the solar spectrum.

The following remarks may be regarded as extensions of the writer's recent paper, "The Theory of Ionization and the Sun-Spot Spectrum."[3] The ideas on which they are based grew largely out of conversations between the writer and Professors K. T. Compton, of Princeton, and F. A. Saunders, of Harvard, to both of whom he desires to acknowledge his indebtedness.

1. *Barium.*—The spectroscopic behavior of this element indicates that it is ionized to a much higher degree in the solar atmosphere, and even in the spots, than would be expected from its ionization potential, which is substantially identical with that of sodium.

The statement in the previous paper, that none of the lines of the neutral barium atom appear in the solar spectrum, is incorrect, for the second line of the principal series at λ 3071 is faintly present, although the first (at λ 5535) is absent or excessively faint, and

[1] *Contributions from the Mount Wilson Observatory*, No. 236.

[2] Research Associate of the Mount Wilson Observatory.

[3] *Mt. Wilson Contr.*, No. 225; *Astrophysical Journal*, **55**, 119, 1922.

King[1] records that a number of lines belonging to the fundamental series of triplets,[2] which are very diffuse in the arc, but sharp in the vacuum furnace, are present in the sun, though very faint. Table I gives a list of all those barium lines which can be identified with tolerable certainty in the solar spectrum. The first three columns

TABLE I

LINES OF BARIUM IN THE SOLAR SPECTRUM

ARC SPECTRUM (KING)			SOLAR SPECTRUM (ROWLAND)				
I. A.	Int.	Class	λ	Origin	Int.	Δλ	Notes
3071.592.....	100 R	III	1.662	Ba	oo	−0.044	(1)
3891.788.....	50	V	1.918	.....	o	− .012	(2)
3995.633.....	30	III	5.769	Ba?	ooo	− .010	(3)
4130.683.....	80	V	0.804	Ba	2	− .036	(2)
4166.027.....	20	V	6.161	Ba	o	− .018	(2)
4524.946.....	35	V	5.110	.....	o	− .004	(4)
4554.038.....	1000 R	II	4.211	Ba	8	+ .003	(5)
4636.333.....	15 N	IV	6.501	.....	o	− .007	.....
4934.099.....	700 R	II	4.214 4.277	Ba	3 4	− .027	(5)
5267.013.....	20	IV?	7.203	.....	ooo	+ .005	.....
5853.699.....	200	III	3.902	Ba?	5	− .009	.....
6141.760.....	600 R	III	1.938	Fe, Ba	7	− .032	.....
6496.902.....	600 R	III	7.128	Fe	4	+0.013	(6)
			Doubtful Lines				
3993.395.....	80	II	3.616	Ba?	oo N	+0.065	(3)
4132.444.....	20	I	2.560	Ba?	o	− .041	.....
4283.111.....	100	II	3.414	Ba?	oo N	+ .141	.....
4350.375.....	80	II	0.550	Ba?	oo N	+ .012	.....
4505.936.....	40	II	6.259	Ba?	oo N	+ .156	.....
4573.881.....	40	II	3.960	Ba?	ooo	− .091	.....
4899.970.....	35	IV?	0.095	Ti, La	2	(− .055)	(4) (7)
6019.505.....	100	II	9.596	Ba?	ooo	− .127	.....
6110.808.....	300 R	II	1.005	.....	oooo	+ .064	.....
6595.351.....	125	I	5.590	.....	oooo	+0.010	.....

NOTES

1. Second member of the principal series of neutral atom 1s–3p.
2. Second member of diffuse series of enhanced lines 2P–4D.
3. Part of first member of fundamental series of neutral atom 3d–4f.
4. First pair of sharp series of enhanced lines 2P–3S.
5. First pair of principal series of enhanced lines 1S–2P.
6. Rowland gives this as an iron line: but the line in question is at 6496.462 (I.A.) according to Burns. The solar line is probably due to barium.
7. The barium line, if present, is masked.

[1] *Mt. Wilson Contr.*, No. 150; *Astrophysical Journal*, **48**, 13–34, 1918.

[2] Saunders, *Astrophysical Journal*, **51**, 25, 1920.

give the wave-length in I.A. from the measures of King or Schmitz, the intensity in the arc, and the temperature class; the next three, Rowland's wave-length, origin, and intensity; and the next, the difference $\Delta\lambda$ between Rowland's wave-length and the laboratory value after the application of St. John's value of the systematic correction.

All the high-temperature lines of King's classes IV and V, which are of intensity greater than 20 in the arc, appear in the solar spectrum, while numerous very strong low-temperature lines of classes I and II and intensities from 200 to 1000, are entirely absent. Most of the lines which are certainly present are well-known enhanced lines. The strong lines at 5853, 6141, and 6496 constitute, according to Saunders,[1] the first member of the diffuse series of enhanced lines, and are analogous to $\lambda\lambda$ 8662, 8542, 8498 of calcium, but reversed in order of wave-length. They are unaffected in the spot spectrum. The enhanced lines of the sharp series appear to be faintly present, and not absent, as stated earlier.

It appears, therefore, that neutral atoms of barium are probably present in the solar atmosphere, in a proportion which, though very small, is not insensible, as was stated in the preceding paper. The conclusion that barium is very highly ionized in the reversing layer, and even in spots, is confirmed by the re-examination of the data.

The conspicuous difference between its behavior and that of sodium remains to be explained. The solution of the puzzle probably lies in the fact that two ionizing agencies are certainly at work in the solar atmosphere—the temperature of the medium, and the radiation poured through it from the hotter photosphere below. Only the first is taken into consideration in the equations of Nernst and the theory of Saha, and, if it alone were active, there would be very strong reason to expect that the percentage of ionized atoms would be the same for the two elements. It is easy to see, however, that the second must create a difference of the sort which is observed.

Consider a barium atom which has just become ionized. Its first (or most easily removable) outer electron is lost; the second will be in the normal state of least energy content, denoted by 1S. In this condition it will immediately recombine with an electron

[1] Unpublished.

which meets it under suitable conditions. But, if photospheric radiation (λλ 4934, 4554) falls upon it, it will absorb energy and the second electron will be lifted into one of the pair of positions 2P. If, after this, the charged atom meets a free electron, it is not so certain what will happen. It may be that it cannot recombine at all unless the second electron is in the normal position. If it can do so, the energy corresponding to the shifting of the second electron and the binding of the first one would have to be radiated. If this was done in a single quantum, the frequency would correspond to a "combination" between terms of the series of ordinary and enhanced lines, and such combinations have never been observed. Leaving aside the more delicate question whether the atom could emit two quanta at once from different electrons, it appears probable in any event that the *Ba*+ atom with its second electron in the 2P state would be *less likely* to recombine than one in the 1S state. Moreover, it will be decidedly easier to ionize a second time, for the energy required to remove the second electron is less (7.15 volts instead of 9.86, after absorption of λ 4554). There is also the probability to be considered that the absorption of a second quantum of radiation may raise the second electron to the state 3D, to which the considerations just urged will apply with even greater force.

The absorption of photospheric radiation by the ionized atoms of barium must therefore discourage recombination and encourage further ionization. Similar absorption by the neutral atoms will assist their ionization, so that, on the whole, the radiation must have a powerful ionizing influence.

For sodium, this influence must be very much smaller, and that for two reasons. In the first place, the enhanced lines of sodium, or at least the fundamental ones, lie in the extreme ultra-violet, beyond λ 377,[1] where the radiation of the photosphere is probably insensible, so that the sodium atom, once ionized, is not further disturbed by sunlight. Secondly, the principal lines of the neutral atom (the D lines) are very strong in the solar spectrum, with wings which indicate that they are absorbed to a great depth. Sodium atoms in the outer parts of the solar atmosphere are

[1] Millikan, *Proceedings of the National Academy of Sciences*, 7, 290, 1921.

therefore exposed to a much enfeebled radiation of just those wavelengths which would be effective in aiding ionization.

For the corresponding lines of barium, on the contrary, the absorption is exceedingly small, and neutral atoms in the upper atmosphere are exposed to the full influence of the radiation.

When all these factors are considered, it is no longer surprising that barium is more highly ionized than sodium, in spite of the equality of their ionization potentials.

Whether the influence of radiation is competent to produce as great an effect as that which is observed can be determined only when a quantitative theory of its action has been developed. It may be noticed, however, that the $_3D-_2P$ lines of $Ba+$ are almost as strong in the solar spectrum as the pair $_1S-_2P$, which indicates that a large fraction of all the singly ionized barium atoms in the reversing layer are in the states of "abnormally" large energy content.

The principle here sketched is evidently generally applicable. An element which has strong enhanced lines in the region where the continuous spectrum of the sun (or, for that matter, of any star) is strong, will behave as if it was relatively easier to ionize than one which does not possess such lines. The behavior of strontium, which resembles barium, but is less conspicuously "anomalous," is thus explained.

Most of the elements which are prominent in the solar spectrum possess such enhanced lines, with the exception of the alkali metals. The spectroscopic method suggested in the former paper should, therefore, give relatively high values for the ionization potentials of these metals, while it is probable that the errors for the other elements will be smaller. The alkaline earths, whose enhanced lines are very favorably situated, will probably come out with ionization potentials too low.

2. *Manganese.*—Saha[1] quotes the value 7.6 volts for the ionization potential, derived from unpublished studies by Catalan upon the series in its spectrum. This agrees satisfactorily with the spectroscopic evidence which puts its ionization potential rather more than halfway from *Ca* (6.1) to *Zn* (9.4).

[1] *Nature*, **107**, 683, 1921.

3. *Rubidium.*—Saunders[1] notes that the strongest of the enhanced lines given by Kayser[2] appear to be present, though faint, in the solar spectrum—the wave-lengths on Rowland's scale being given in Table II.

TABLE II

ENHANCED LINES OF RUBIDIUM

SPARK (EXNER AND HASCHEK)		SUN (ROWLAND)	
λ	Int.	λ	Int.
3940.62	20	Absent	
4104.49	10	4104.459	00
4244.50	30	4244.500	00
4294.08	10	4294.077	000

The evidence for the presence of this metal in the sun is therefore strengthened. The principal enhanced lines lie probably in the far ultra-violet.

None of the stronger enhanced lines of the other alkali metals, as given in Kayser's *Handbuch*, appear to be present in Rowland's tables.

4. *Lithium.*—The absence of lithium lines from the solar spectrum, and their faintness in spots, still presents much difficulty. Lithium behaves almost exactly like rubidium, though its ionization potential is 5.37 volts, and that of the latter 4.16.

We cannot invoke any absorption of enhanced lines here, for none have even been discovered.[3] Compton suggests[4] that one factor in the problem is the small atomic weight. This will cause the mean atomic velocity of temperature agitation to be greater than for other elements (about three and one-half times that for rubidium), and so broaden out all its lines, so that the atoms are able to take hold (so to speak) of a correspondingly greater range of wave-length in the photospheric radiation, and use it in assisting their ionization.

What other factors may enter remains for future discovery.

MT. WILSON OBSERVATORY
February 22, 1922

[1] Not previously published.

[2] *Handbuch der Spectroskopie*, **6**, 350.

[3] Millikan, *op. cit.*, p. 292.

[4] Unpublished.

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THE EDITORS